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Demonstration of phase transitions using a 1D transverse mechanical topological insulator

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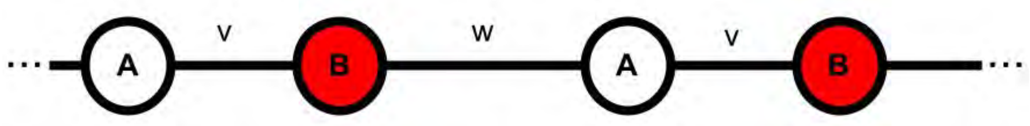
INTRODUCTION

Topological insulators are a type of material that allows the existence of unidirectional currents at the quantum scale. These currents, called edge states, are unaffected by material imperfections, which makes topological insulators an active research topic with potential applications in quantum computing. Protected edge states - analogous to the unidirectional current of electronic topological insulators - have recently been demonstrated not only with electrons, but also with photons, sound waves, and mechanical waves. We constructed a one-dimensional mechanical model of topological insulators that uses transverse waves in an elastic string. We based our design on a theoretical model known as the SSH model [1]. Our device demonstrated a phase transition between the insulator, conductor, and topological insulator phases of the SSH model, with clearly defined edge states in the topological insulator phases.

SSH MODEL

The Su-Schrieffer-Heeger (SSH) model describes a one-dimensional diatomic chain with alternating electron hopping probabilities *v* and *w*.



The relative values of *v* and *w* determine if the material is an insulator, conductor, or topological insulator as follows:

Relationship	Phase	Shorthand Name
v > w	Insulator	“Trivial”
v = w	Conductor	“Metallic”
v < w	Topological Insulator	“Topological”

Table 1: *v* and *w* values corresponding to various phases of the SSH model.

In our mechanical model, we use transverse elastic waves traveling through a string instead of electrons. In this case *v* and *w* represent the strengths of the interactions from A to B and from B to A, respectively.

METHODS

EXPERIMENTAL

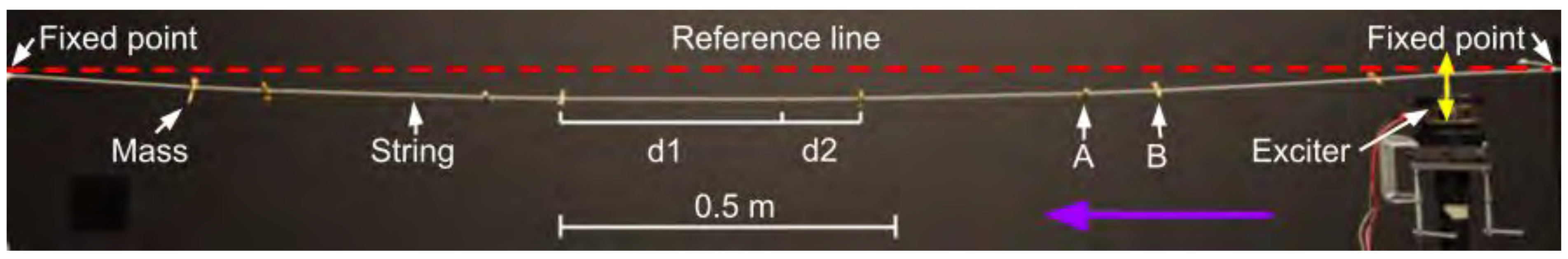


Figure 1: Experimental setup.

Using the above setup, we excited an elastic string at controlled frequencies and recorded the frequencies corresponding to resonant modes. To change configurations between trivial, metallic, and topological, we changed the locations of the metal masses.

d1 (cm)	*d2* (cm)	*d1*/*d2*	Phase
11.2	33.8	1/3	Trivial II
15.0	30.0	1/2	Trivial I
22.5	22.5	1	Metallic
30.0	15.0	2	Topological I
33.8	11.2	3	Topological II

Table 2: *d1* and *d2* values corresponding to various phases of the transverse mechanical model.

THEORETICAL

We developed our own theoretical model of our device using differential equations of motion.

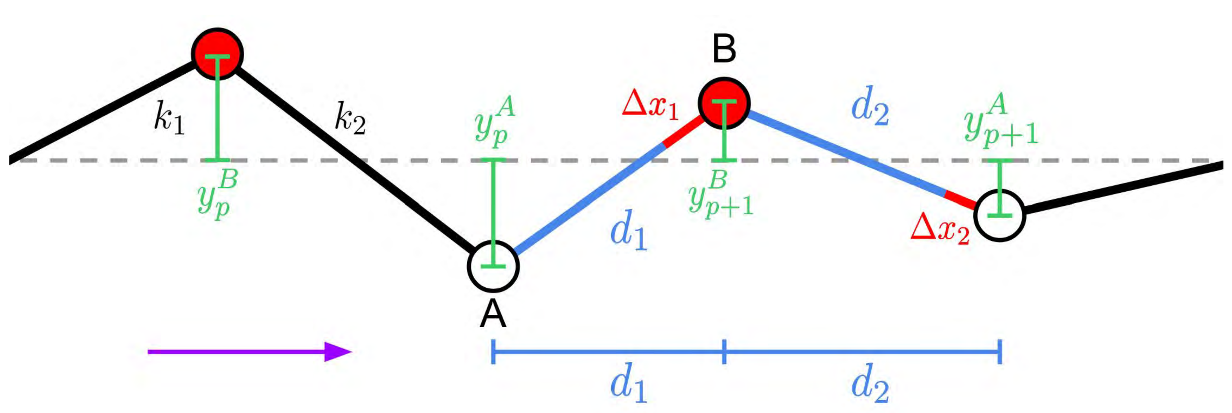


Figure 2: Diagram of the transverse mechanical model with variables for the theoretical model labeled.

The differential equations of motion corresponding to our system are:

$$\ddot{y}_p^A + \alpha\left(y_p^A - y_{p+1}^B\right) + \beta\left(y_p^A - y_p^B\right) = 0,$$
$$\ddot{y}_p^B + \alpha\left(y_p^B - y_{p-1}^A\right) + \beta\left(y_p^B - y_p^A\right) = 0. \quad (1)$$

Here $\alpha = \frac{k_1 \Delta x_1}{m(d_1+\Delta x_1)}$ and $\beta = \frac{k_2 \Delta x_2}{m(d_2+\Delta x_2)}$. These equations describe each of our 10 masses with a 10x10 eigenvalue problem:

$$\begin{pmatrix} \alpha+\beta & -\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\beta & \alpha+\beta & -\alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha & \alpha+\beta & -\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta & \alpha+\beta & -\alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha & \alpha+\beta & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\beta & \alpha+\beta & -\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\alpha & \alpha+\beta & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\beta & \alpha+\beta & -\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & \alpha+\beta & -\beta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta & \alpha+\beta \end{pmatrix} \mathbf{y} = \omega^2 \mathbf{y}, \; \mathbf{y} = \begin{pmatrix} y_1^B \\ y_1^A \\ y_2^B \\ y_2^A \\ y_3^B \\ y_3^A \\ y_4^B \\ y_4^A \\ y_5^B \\ y_5^A \end{pmatrix}. \quad (2)$$

Solving for the eigenvalues of this equation gives us values that we can convert to frequency and compare to our experimental results. Additionally, we ran computer simulations of our setup using the software COMSOL Multiphysics.

RESULTS

We demonstrated a band gap in the trivial II case, Fig. 3(a), that decreased in magnitude for the trivial I case, Fig. 3(b), and disappeared for the metallic case, Fig. 3(c). The band gap reopened for the topological I case with two edge modes within the band gap, Fig. 3(d), and widened again for the topological II case, preserving the two edge modes, Fig. 3(e).

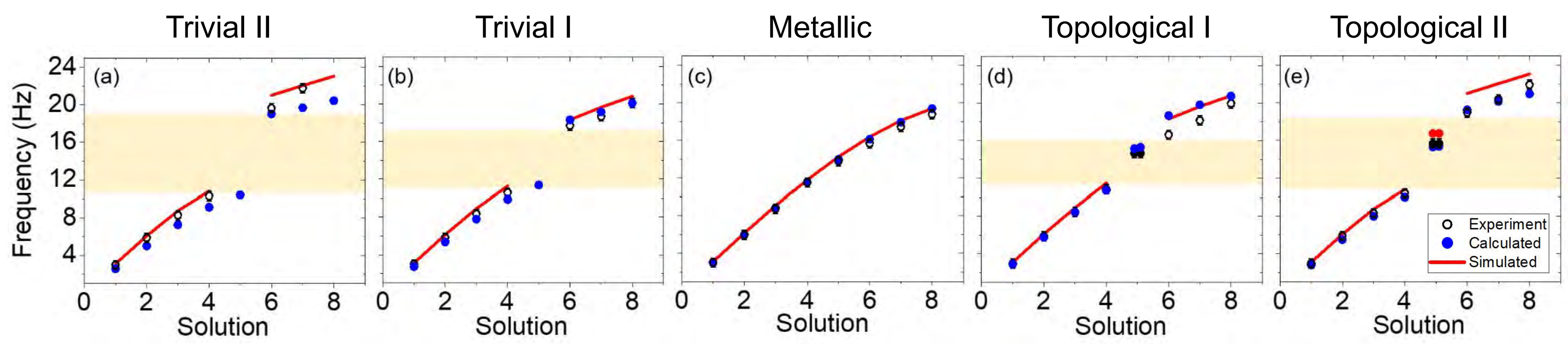


Figure 3: Experimental, theoretical, and simulated resonant frequencies for the 5 phases we studied. The shaded yellow regions represent the experimentally observed band gaps.

The experimental and simulated modes of the topological II case are shown in Fig. 4. We observed two edge modes, localized at the left, Fig. 4(e), and right, Fig. 4(f), of the device.

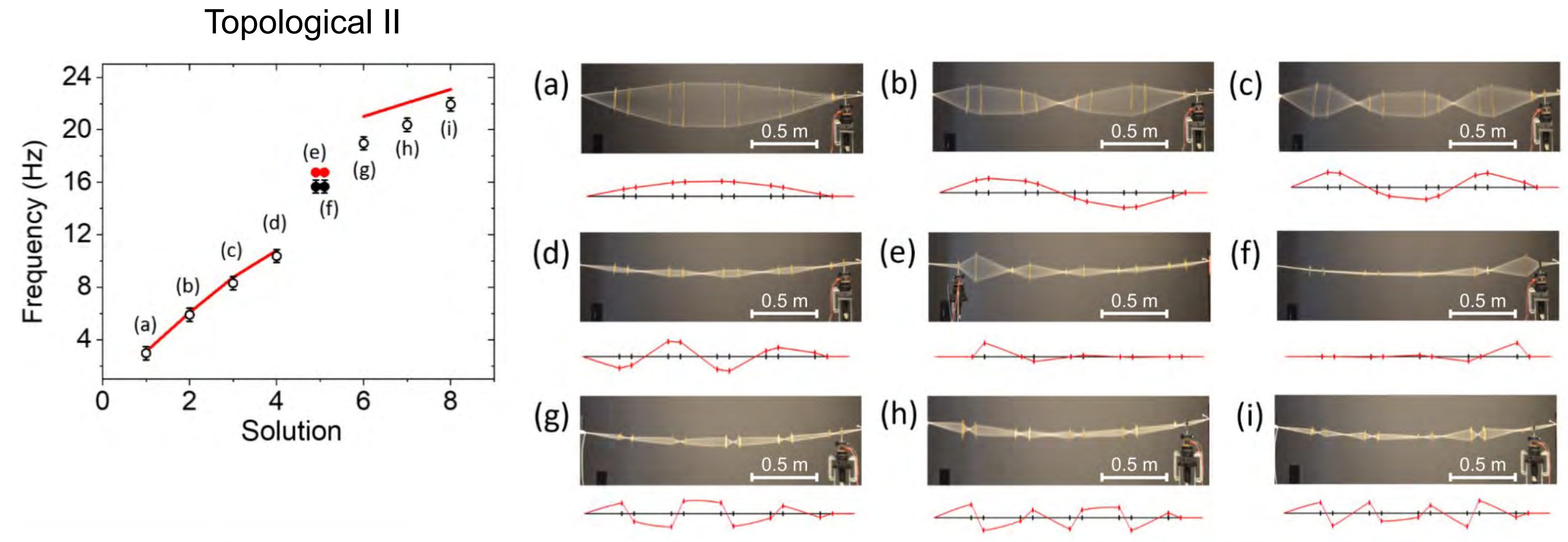


Figure 4: Experimental and simulated results from Fig. 3(e) with the experimental and simulated modes pictured on the right.

CONCLUSION

Using an elastic string with masses in a crystal array, we have demonstrated phase transitions between the trivial, metallic, and topological configurations of the 1D-SSH model. In the topological I and topological II configurations, we observed edge states localized at the extremes of our device. Our findings are supported by numerical simulations and a theoretical model derived from the equations of motion of our system.

In future work, we hope to continue this study on two-dimensional mechanical systems.

REFERENCES + ACKNOWLEDGEMENTS

[1] W. P. Su, J. R. Schrieffer, and A. J. Heeger, “Solitons in polyacetylene,” Phys. Rev. Lett. 42, 1698-1701 (1979).

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