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Demonstration of phase transitions using a 1D longitudinal mechanical topological insulator

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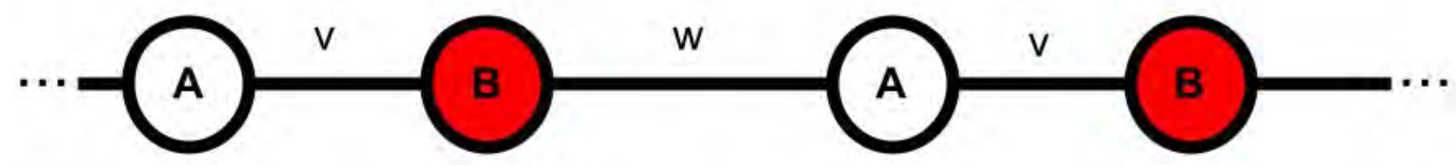
INTRODUCTION

Topological insulators are a type of material that allows the existence of unidirectional currents at the quantum scale. These currents, called edge states, are unaffected by material imperfections, which makes topological insulators an active research topic with potential applications in quantum computing. Protected edge states - analogous to the unidirectional current of electronic topological insulators - have recently been demonstrated not only with electrons, but also with photons, sound waves, and mechanical waves. Using a theoretical model of topological insulators known as the SSH model, we constructed a one-dimensional mechanical model of topological insulators that uses longitudinal waves in a metallic spring.



SSH MODEL

The Su-Schrieffer-Heeger (SSH) model describes a one-dimensional diatomic chain with alternating electron hopping probabilities v and w.

Relationship	Phase	Shorthand Name
v > w	Insulator	“Trivial”
v = w	Conductor	“Metallic”
v < w	Topological Insulator	“Topological”

In our mechanical model, we use longitudinal waves traveling through a spring instead of electrons. In this case v and w represent the strengths of the interactions from A to B and from B to A, respectively.

METHODS

EXPERIMENTAL

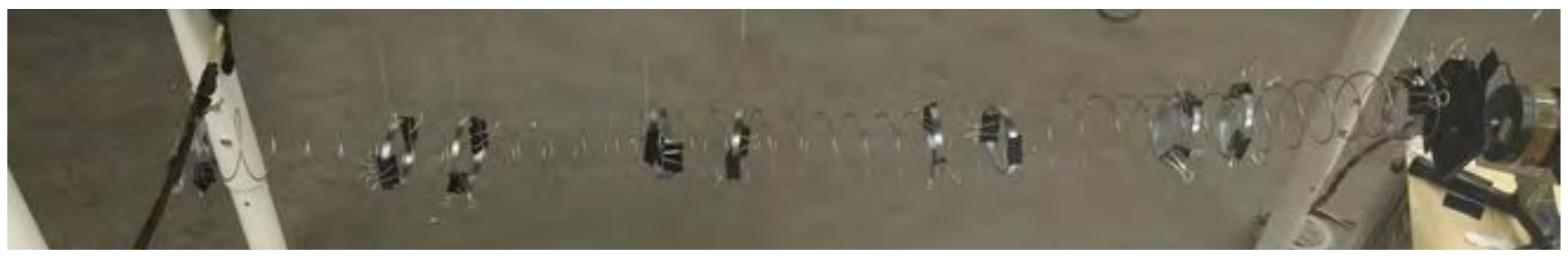


Fig. 1: Experimental setup.

Using the above setup, we excited a metal spring at controlled frequencies and recorded the frequencies corresponding to resonant modes. To change configurations between trivial, metallic, and topological, we changed the spacing between spring groups.

THEORETICAL

1 Mechanical 1D-SSH Model: Longitudinal

$$m\ddot{x}_p^A + k_1\left(x_p^A - x_{p+1}^B\right) + k_2\left(x_p^A - x_p^B\right) = 0 \quad (1)$$
$$m\ddot{x}_p^B + k_1\left(x_p^B - x_{p-1}^A\right) + k_2\left(x_p^B - x_p^A\right) = 0 \quad (2)$$

Assume $x_p^A = A_p e^{-i(\omega t - \kappa\Lambda p)}$, $x_p^B = B_p e^{-i(\omega t - \kappa\Lambda p)}$.

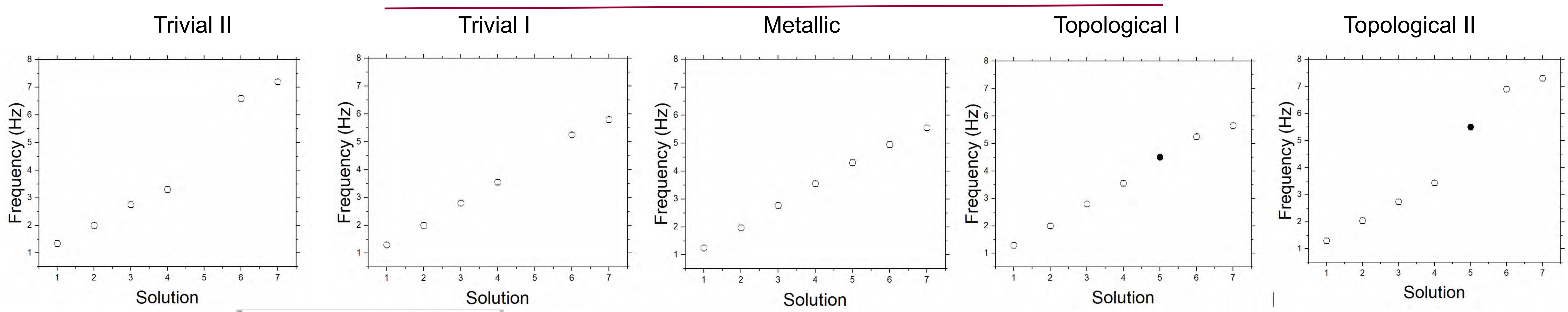
$$x_p^B\left(k_1 + k_2\right) + x_p^A\left(-k_1 e^{-i\kappa\Lambda} - k_2\right) = \omega^2 m x_p^B \quad (3)$$
$$x_p^B\left(-k_1 e^{i\kappa\Lambda} - k_2\right) + x_p^A\left(k_1 + k_2\right) = \omega^2 m x_p^A \quad (4)$$

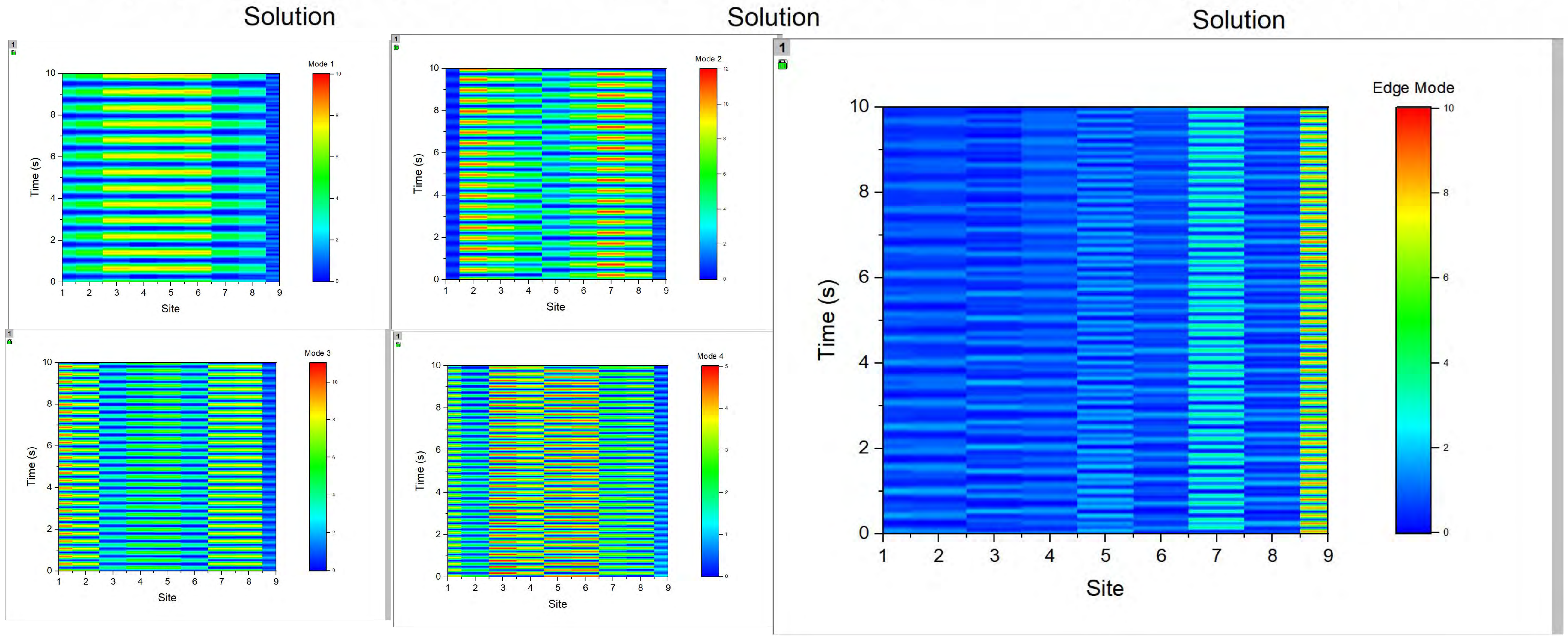
$$\begin{pmatrix} k_1 + k_2 & -k_1 e^{-i\kappa\Lambda} - k_2 \\ -k_1 e^{i\kappa\Lambda} - k_2 & k_1 + k_2 \end{pmatrix}\begin{pmatrix} x_p^B \\ x_p^A \end{pmatrix} = \omega^2 m \begin{pmatrix} x_p^B \\ x_p^A \end{pmatrix} \quad (5)$$

$$H = \begin{pmatrix}
k_1+k_2 & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-k_2 & k_1+k_2 & -k_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -k_1 & k_1+k_2 & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -k_2 & k_1+k_2 & -k_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -k_1 & k_1+k_2 & -k_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -k_2 & k_1+k_2 & -k_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -k_1 & k_1+k_2 & -k_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -k_2 & k_1+k_2 & -k_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_1 & k_1+k_2 & -k_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_2 & k_1+k_2
\end{pmatrix}$$

$$H\mathbf{x} = \omega^2 m\mathbf{x}, \quad \mathbf{x} = \begin{pmatrix} x_1^B \\ x_1^A \\ \vdots \\ x_5^B \\ x_5^A \end{pmatrix} \quad (6)$$

RESULTS





CONCLUSION

Using a metallic spring with purposefully spaced groupings in a repeating lattice, we have demonstrated phase transitions between the trivial, metallic, and topological configurations of the 1D-SSH model. In the two topological configurations that we studied, we observed edge states localized at the extremes of our device. Our findings are supported by numerical simulations and a theoretical model derived from the equations of motion of our system.In future work, we hope to continue this study on two-dimensional mechanical systems, and demonstrate edge states along each side and each corner of our device.

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