Bargaining in the shadow of precedent: the surprising irrelevance of asymmetric stakes

Alan C. Marco — Vassar College

Kieran Walsh — Vassar College

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Abstract

We develop a model of bargaining and litigation in the context of patent licensing (or any contractual setting). Following Priest and Klein (1984) we developed a model that explicitly allows for (1) multiple parties (leading to asymmetry of stakes), (2) binding precedent, and (3) pre-dispute bargaining done in the "shadow" of precedent-setting courts. The pre-dispute bargaining creates an endogenous opportunity cost of litigation for both plaintiff and defendant; i.e., the harm is endogenous. We show that the effects of asymmetric stakes on the litigation rate and plaintiff win rate are offset by opportunity costs (forgone licensing). That is, the degree of asymmetry does *not* appear to substantially impact the rate of litigation or the observed win rate of plaintiffs at trial. This result is in stark contrast to the previous theoretical literature, and has implications for interpreting the empirical literature.

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1 Introduction

Since the seminal work of Priest and Klein in 1984 (hereafter, PK) scholars have been aware of the importance of selection effects in the study of litigated disputes. Their model, and subsequent contributions by other scholars (Cooter and Rubinfeld 1989, Che and Yi 1993, Waldfogel 1995, Daughety and Reinganum 1993), show formally that litigated disputes are pathological, and that the set of disputes selected for litigation are not representative of the population of disputes. The most fundamental result of PK is that the plaintiff win rate of litigated cases will be biased towards 50% relative to the population win rate. The bias toward 50% can be either upward or downward depending on whether the population win rate is higher or lower than 50%. As the litigation rate becomes smaller, the bias will be more acute because the selection effects become stronger.

As noted by PK, the 50% rule is not robust to all specifications—notably asymmetric stakes. Asymmetry is an important extension in many instances, since it represents the case where either (1) the plaintiff has more to gain than the defendant has to lose; or, (2) the defendant has more to lose than the plaintiff has to gain. The most straightforward justification for this scenario is that of precedent. Cited examples include products liability (PK), medical malpractice (Wittman 1988), and tobacco trials (Che and Yi 1993). In the recent Merck Vioxx trials, a large defendant faces many potential plaintiffs in similar suits. The outcome of the initial trial has ramifications beyond the potential damage payment from defendant to plaintiff. In that instance, the stakes for the defendant are much larger (in dollar value) than those of the plaintiff.

In the case of patents a patent holder bringing an infringement suit against an alleged infringer may find its bargaining position with licensees improved as the result of an infringement verdict (Waldfogel 1995, Lanjouw and Schankerman 2001). Further, the patent holder risks invalidation of the patent. Obviously any such invalidation may have consequences that go beyond the current dispute.

In contrast to the symmetric stakes finding, PK find that greater asymmetry can radically change the results of their model. In fact, the bias in the win rate due to selection can go the opposite direction (away from 50%; see figure 1). Waldfogel (1995) finds evidence of this in intellectual property disputes.

The case of asymmetric stakes relates more generally to any situation where precedent is more important to one party than to another. Che and Yi (1993) investigate precedent in a two period model with two identical plaintiffs. They point out that a party's decision to go to court will include facing potential gains or losses from trial, as well as the precedent set for future cases.[1] Parties will alter their pre-trial bargaining strategies in the face of precedent.

Like Che and Yi (1993), the current paper allows for both good and bad precedent. However, the modeling strategy follows PK very closely, in order to facilitate a comparison of the conclusions about selection and litigation. Additionally, we allow asymmetry to be defined broadly with an arbitrary number of heterogeneous opponents.

PK consider only the asymmetric stakes related to a plaintiff win. That is, they examine only the "up side" to asymmetry. For instance, in PK's model it is assumed that the value of the judgment to a winning plaintiff is J. With asymmetric stakes, the defendant—if it loses—pays αJ, where $\alpha \in (0, 1]$. The plaintiff receives αJ from the defendant and $(1 - \alpha)J$ in the form of some value of precedent. If the plaintiff loses, PK assume that the status quo prevails (with the status quo being 0).[2] Figure 1 replicates a simulation from PK, where the horizontal axis is the level of symmetry between plaintiff and defendant, α. One can see that with symmetric stakes ($\alpha = 1$), the observed win rate is biased towards 50% relative to the population win rate. As asymmetry increases ($\alpha < 1$), the litigation rate increases and the observed win rate actually rises above the population win rate, i.e., the 50% bias is reversed.[3]

Generalizing PK, we develop a model of litigation in which we account for the opportunity cost of litigation. The model explicitly allows for (1) multiple parties (leading to asymmetric stakes), (2) binding precedent, and (3) pre-dispute bargaining done in the "shadow" of precedent. The pre-dispute bargaining creates an endogenous opportunity cost of litigation for both plaintiff and defendant.[4] Including the opportunity cost of litigation enables us to examine both the up side and down side of asymmetry. We examine both the win rate and the litigation rate in the face of endogenous opportunity costs, and find striking contrasts with PK's results. The most striking result is that asymmetric stakes do not generally alter the litigation condition relative to the symmetric stakes case. This result is important for the empirical literature relying on the asymmetric stakes interpretation of PK. We show that this result, previously presumed in much of the empirical literature, is a special case.

[1] Technically, Che and Yi do not model precedent *per se*; rather, they assume that cases are correlated. Parties become more informed about the legal standard for correlated cases when the court rules on the first case.

[2] PK themselves point out the limitation of the assumption of a simple J payoff. To demonstrate the basic principle of selection and its effects on win rates, this assumption is innocuous.

[3] The figure assumes that the total legal costs of the two parties is equal to $\frac{1}{3}J$, and that the standard deviations of the plaintiff's and defendant's errors are 0.3 (see section 2 for more detail).

[4] In a broader sense the endogenous opportunity cost endogenizes the *harm* that will arise from any future dispute.

The next section develops the basic structure of the model in the context of patent litigation. We introduce the concept of a pre-dispute royalty rate, which represents the opportunity cost of litigation. Section 3 extends the model to include multiple defendants (from which we derive the asymmetry from precedent). The pre-dispute equilibrium r is the result of implicit bargaining between the patent holder and all potential licensees. We derive the litigation condition and compare the results to those of PK. Section 5 discusses the implications of the model, and applies the analysis to legal setting where parties can bargain (or, otherwise take measures to reduce exposure to litigation) prior to the dispute. We argue that endogenous opportunity costs and asymmetry are the general case, of which PK represents a special case.

Pre-dispute bargaining creates endogenous harm by the licensees/infringers, in that infringement on a higher r will lead to greater lost profits to the plaintiff than infringement on a lower r. The bargaining can be implicit or explicit, in the sense that rational infringing parties will recognize the lost opportunity to license at the going royalty rate when they infringe. In the model, infringers presume that the royalty rate would be negotiated in the shadow of threats about litigation, the consequences of precedent, and beliefs about validity. The model has applications beyond patent law, to any area of law where potential defendents are subject to precedent, and where harm can be mitigated through investment in precaution.

2 Model

The model is motivated by intellectual property litigation. We use patent litigation as an example because it concretely applies the notions of pre-dispute bargaining (endogenous opportunity cost) and asymmetry. The opportunity cost of litigation is in the form of a hypothetical license that each party rejects in favor of litigation. The asymmetry of stakes in the model relates to patent validity. If a patent holder determines to prosecute an infringement claim, it risks invalidation of the patent. Invalidation means a lost opportunity to receive royalty income from other licensees, or damages from other infringers. On the other hand, a valid patent means greater bargaining power licensees and infringers alike. In this way the asymmetry has both an up side and a down side. Bargaining over the hypothetical license proceeds strategically.

Following PK, let $Y'\tilde{}N(0,1)$ be a random variable representing the facts of individual cases. y' is an individual draw of Y'. In the patent context, the draw of y' represent the facts surrounding the validity of a patent.

Additionally, Y^* represents the decision standard. If $y' > Y^*$ then the patent "quality" exceeds the patentability bar, and the patent is valid. However, both the patent holder and potential infringers/licensees measure y' with error. In particular, $\hat{Y}_A = Y' + \varepsilon_A$ is a random variable representing the patent holder's guess regarding patent quality, where ε_A is distributed $N(0, \sigma_A)$. Similarly, $\hat{Y}_B = Y' + \varepsilon_B$ is the potential infringer's estimate of patent quality. Where ε_A and ε_B are distributed with density functions f_A and f_B, and with cumulative distribution functions F_A and F_B, respectively.

Given draws $\widehat{y}_A$ and $\widehat{y}_B$ we can define A's and B's beliefs about patent validity as

$$p_A = \Pr(Y' > Y^* | \widehat{y}_A) = F_A(\widehat{y}_A - Y^*) \tag{1}$$

$$p_B = \Pr(Y' > Y^* | \widehat{y}_B) = F_B(\widehat{y}_B - Y^*). \tag{2}$$

Again, following PK, we consider a dispute between parties A and B, regarding A's patent. The value of the patent to B is given by v. If B is to use the technology, the parties may elect to bargain over a licensing contract at royalty r, where $r \in [0, 1]$ indicating a proportion of the total value of the innovation to B. The total royalty payment is rv. If they forgo licensing, then they litigate. In litigation, each pays litigation costs. If $y' > Y^*$ then the court rules the patent valid, and the patent holder receives v from B. If $y' < Y^*$, then A loses the patent right, and the patent lapses into the public domain (in which case B may infringe the technology with impunity). If the parties license, their payoffs are

$$\pi_A = rv \tag{3}$$

$$\pi_B = (1 - r)v \tag{4}$$

The expected gross payoff that each party believes it will receive from litigation are given by

$$\pi_A = p_A v - c_A \tag{5}$$

$$\pi_B = (1 - p_B)v - c_B, \tag{6}$$

where c_A and c_B are the litigation costs for each party.[5] These expected payoffs mirror those used by PK. However, when each party litigates, it gives up the opportunity to license at some royalty r, expressed as a proportion of the value v paid by B to A. For an arbitrary r, A's net benefit of litigation (or the "ask" price to settle the dispute) can be written as

$$a = p_A v - c_A - rv, \tag{7}$$

[5] For parsimony, we assume that settlement costs are zero; or, alternatively, that c represents the difference between litigation costs and settlement costs. The qualitative results are robust to this assumption.

where r represents the opportunity cost of giving up the opportunity to license. This is the minimum payment that A will accept to forgo litigation. Similarly, B's bid to settle reflects the cost of giving up the opportunity to license,

$$b = p_B v + c_B - rv. \tag{8}$$

The value of b reflects the maximum amount that B is willing to pay to forgo litigation (it obtains $(1-r)v$ from settling, and loses $(1-p_B)v - c_B$ by going to court).

These bid and ask prices reflect those of PK except for the opportunity cost rv.

A sufficient condition for litigation to occur is if $a > b$,[6] or

$$p_A v - c_A - rv \quad > \quad p_B v + c_B - rv \tag{9}$$

$$p_A - p_B \quad > \quad \frac{c_A + c_B}{v}. \tag{10}$$

The condition is equivalent to saying that there is no cooperative surplus to settlement, where cooperative surplus is given by $b - a$.

In the case of symmetric stakes, the opportunity cost r drops out of the litigation condition, and the condition reduces to the standard PK result. However, when there are multiple potential infringers/licensees, the patent holder is faced with asymmetry of stakes.

3 Asymmetric stakes

We consider asymmetry only on the part of the patent holder, A. That is, we consider a patent holder facing N potential licensees/litigants. The asymmetry arises in this case because of precedent. If the patent holder tries one case against a single infringer, then we assume that the ruling on validity will be binding on all future cases involving the same technology. So, once a patent is validated, the precedent is binding and validity is known with certainty and can be enforced costlessly. If a patent is invalidated, then the patent lapses into the public domain and all licensees/infringers may use the technology without fear of reprisal. Assuming that proven validity affects bargaining power, then the precedent will enable the patent holder to begin "knocking on the doors" of other licensees/infringers in order to secure a higher royalty.

Formally, we assume N licensees/infringers indexed by $B_1 ... B_N$. The technological value of the patent remains v. We can think of licensee i as having a market share α_i, so that the private value

[6] It is possible for bargaining to break down even if $a < b$. However, if there is no cooperative surplus, then it will be impossible for the parties to bargain to a mutually agreeable settlement. Thus, the litigation condition given here is sufficient.

of the patent to licensee i is $\alpha_i v$, and $\sum_{i=1}^{N} \alpha_i = 1$. That is, v is the total fixed technological value of the patent distributed according to the market shares of the producers in the market.[7] In any particular negotiation between A and B_i, α_i represents the degree of asymmetry. This is because the court's decision about validity applies to the patent as a whole, not only the α_i portion used by B_i. Thus, the patent holder has greater stakes than the licensee/infringer.[8]

In bargaining, each pair A and B_i will consider a royalty rate r_i against which they will consider the litigation decision. As in the single licensee case, r_i is a royalty rate relative to the patent value v. So, the total royalty payment between A and B_i is $\alpha_i r_i v$, which reflects the opportunity cost of litigating. To complete the notation, let c_A be the litigation cost of the patent holder, as above, and let c_i be the litigation cost for licensee i. For notational simplicity let

$$\mathbf{a} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \end{bmatrix}, \ \mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix}, \ \mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix}.$$

3.1 Litigation

Assuming that litigation (if it occurs) occurs only between A and B_i,[9] the payoffs to A and B_i from licensing are

$$\pi^A_{license} = \mathbf{r}'\mathbf{a}v \tag{11}$$

$$\pi^B_{license} = (1 - r_i)\alpha_i v \tag{12}$$

where $\mathbf{r}'\mathbf{a}v$ is the sum of the patentholder's royalty payments from all parties. Note that licensing assumes that all licensees agree on their respective royalty rates. The expected gross payoffs the

[7] The assumption that v and N are fixed may not appropriate for certain kinds of market structures, e.g., free entry Cournot with homogeneous products. However, in the case of a differentiated products market, the assumption is more reasonable. In any case it does not drive the results.

[8] Priest and Klein allow for the asymmetry to favor the plaintiff or defendent. In their parlance, the share of the judgment J that is paid by the defendent to the plaintiff is αJ. Note that α can be greater than one, indicating that the plaintiff receives less than the defendent pays. This might arise in the case of a large defendent facing many possible plaintiffs, e.g., products liability. As discussed in Waldfogel (1995), the asymmetry in the case of patents is likely to favor the patent holder.

In the present case, the fact that asymmetry can only go in one direction does not systematically alter the results, as we show below.

[9] I show below that this assumption holds in equilibrium.

parties believe they will receive from litigation are given by

$$\pi^A_{litigate} = p_A v - c_A \tag{13}$$

$$\pi^B_{litigate} = (1 - p_B)v - c_B. \tag{14}$$

Again, litigation involves an opportunity cost. The patent holder risks invalidating the patent in court. In the case of multiple licensees, the invalidation applies to all parties. So, a valid patent will set a precedent that holds for all future cases, and all licenses will be updated to a royalty rate of $r_i = 1$ for all i, and $\mathbf{r}'\mathbf{a}v = v$. However, and invalid patent implies that the royalty rate will be updated to $r_i = 0$ for all i, and $\mathbf{r}'\mathbf{a}v = 0$. Thus, A's net benefit of litigation (or the "ask" price to settle the dispute) can be written as

$$\begin{aligned} a(\mathbf{r}) &= p_A v - c_A - \mathbf{r}'\mathbf{a}v & (15)\\ &= p_A v - c_A - r_i\alpha_i v - q_i(\mathbf{r})v & (16) \end{aligned}$$

where

$$q_i(\mathbf{r}) = \mathbf{r}'\mathbf{a} - r_i\alpha_i. \tag{17}$$

Similarly, B_i's bid price is given by

$$b_i(r_i) = p_i\alpha_i v + c_i - r_i\alpha_i v. \tag{18}$$

where p_i represents B_i's estimate that the patent is valid.

Litigation will occur if the cooperative surplus is negative, or equivalently if

$$\begin{aligned} a(\mathbf{r}) &> b_i(r_i) & (19)\\ p_A v - c_A - r_i\alpha_i v - q_i(\mathbf{r})v &> p_i\alpha_i v + c_i - r_i\alpha_i v & (20)\\ p_A - \alpha_i p_i &> q_i(\mathbf{r}) + \frac{c_A + c_i}{v}. & (21) \end{aligned}$$

This condition is similar to the PK asymmetric stakes result with the exception of the $q_i(\mathbf{r})$ term. Treating $q_i(\mathbf{r})$ as exogenous means that litigation will become less likely than in the PK setting. However, it is difficult to believe that $\mathbf{r}$ is independent of the beliefs of the parties about validity. We turn in the next section to endogenizing $\mathbf{r}$. Prior to that, we elaborate on the litigation condition.

PK use condition 10 to describe a sufficient condition for litigation. Because allow only for the cases of litigation and settlement, this condition suffices. However, if the parties can bargain on r_i prior to litigation, then a further condition must hold in order for there to be a legitimate dispute. In particular, it must be that

$$a(\mathbf{r}) \geq 0 \geq b_i(r_i) \tag{22}$$

in order for litigation to occur. That is, both parties must prefer litigation to accepting the proposed license agreement in order for litigation to be a rational outcome of the bargaining process. For instance, if $a(\mathbf{r}) < 0$ and $b_i(r_i) < 0$, but $a(\mathbf{r}) > b_i(r_i)$ then we can hardly imagine that litigation will occur. In this case, there is a cooperative surplus to litigation, but only one party (B_i) wants to litigate. This case does not apply to PK because the status quo is assumed to be zero; the parties do not give up the license in order to litigate. Rather, PK assume that the "harm" has occurred and that it is exogenous to the litigation/settlement decision.

In the present model, pre-trial bargaining on the royalty rate is assumed to depend upon the likelihood of going to court and on the beliefs about the parties (or the other model parameters). So, if $a(\mathbf{r}) < 0$, one has to wonder whether the initial bargaining on $\mathbf{r}$ was efficient. Below, we solve for the renegotiation-proof $\mathbf{r}^*$ and show that condition 21 implies 22.

3.2 Settlement and Renegotiation

3.2.1 Settlement

The patent holder and licensee i settle if

$$a(\mathbf{r}) \leq b_i(r_i) \tag{23}$$

and

$$a(\mathbf{r}) \leq 0 \leq b_i(r_i) \tag{24}$$

When these conditions hold, cooperative surplus exists, where the cooperative surplus is given by $b_i(r_i) - a(\mathbf{r})$. In this instance both parties prefer the status quo—the existing royalty scheme—to litigation. No further negotiation or money exchange is required since the initial conditions in effect already divide the cooperative surplus.

Thus, in our model settlement is equivalent to there being no dispute to begin with. A dispute arises if no $\mathbf{r}$ can be found that is agreeable to all parties.

This departs from PK because they assume an exogenous harm, and a dispute is created from that harm. In the patent context if the parties can agree on r, then no harm occurs (no infringement). This does not imply that in all cases no dispute will arise at all. However, we assume that if a dispute arises, that settlement costs are small relative to litigation costs, and that the parties will eventually arrive at an r that would have precluded a dispute to begin with. The next section discusses this bargaining.

3.2.2 Renegotiation

When

$$a(\mathbf{r}) \leq b_i(r_i) \tag{25}$$

then cooperative surplus exists, and the parties will be able to bargain to a settlement. But, if

$$b_i(r_i) < 0 \text{ or } a(\mathbf{r}) > 0 \tag{26}$$

then one party prefers litigation to the status quo. The existing royalty scheme would lead to litigation if the parties were not able to negotiate over the cooperative surplus, because one party would initiate a suit. PK's model (and ours) assumes that if litigation can be avoided then it will be, if settlement costs are low enough (we normalize settlement costs to zero). Conversely, having no cooperative surplus is a sufficient condition for litigation.

To avoid litigation one party must make a side payment to the other to not go to trial. As long as a cooperative surplus exists, then some side payment will be possible. The parties split the surplus by negotiating a new royalty. We assume that renegotiation occurs according to the Nash bargaining solution.[10] In particular, we allow the patent holder to have bargaining power relative to B_i based on the size of α_i. The cooperative surplus of $b_i(r_i) - a(\mathbf{r})$ will be divided between A and B_i according to the shares $\frac{1}{1+\alpha_i}$ and $\frac{\alpha_i}{1+\alpha_i}$, respectively.

When A and B_i renegotiate, the gross payoffs are given by the payoff from litigation plus the share of cooperative surplus. So, after renegotiation the new ask and bid will be

$$a'(\mathbf{r}) = a(\mathbf{r}) + \frac{1}{1+\alpha_i}\left(b_i(r_i) - a(\mathbf{r})\right) \tag{27}$$

$$b_i'(\mathbf{r}) = b_i(r_i) + \frac{\alpha_i}{1+\alpha_i}\left(b_i(r_i) - a(\mathbf{r})\right). \tag{28}$$

After renegotiation, a' and b_i' will lead to settlement whenever $a(\mathbf{r}) \leq b_i(r_i)$. Renegotiation will always cause $a' < 0$ and $b_i' > 0$ whenever $a < b_i$, indicating that neither party wishes to litigate (see proposition 4 and corollary 6).

The gross payoffs after renegotiation will be

$$\pi^A_{renegotiate} = p_A v - c_A + \frac{1}{1+\alpha_i}\left(b_i(r_i) - a(\mathbf{r})\right) \tag{29}$$

$$\pi^i_{renegotiate} = (1 - p_i)\,\alpha_i v - c_i + \frac{\alpha_i}{1+\alpha_i}\left(b_i(r_i) - a(\mathbf{r})\right) \tag{30}$$

[10] Daughety and Reinganum (1993) discuss the timing of litigation models. Like PK we ignore the sequencing of the bargaining game, and rely on the Nash bargaining solution as a simplification. Daughety and Reinganum find that some situations with asymmetric information lead to offers made for the purposes of signalling and screening. However, sequencing of offers never occurs when the timing of offers is endogenous.

Note that the renegotiated royalty rates will depend upon the royalties being paid by all other licensees/infringers because A's opportunity cost is implicitly a function of the average royalty $\mathbf{r'a}$.

3.3 The renegotiation-proof license

When the patent holder and a defendant renegotiate to avoid litigation, the gross payoffs imply an implicit royalty that would have avoided the dispute altogether had it been the original royalty. Since we assume that the parameters of the model are known prior to bargaining, rational parties will negotiate to this royalty in the first instance.

In the case of a single dispute, the gross payoffs to the parties can be written as a function of a new implicit royalty rate. Let $\widetilde{r_i}$ indicate the implicit renegotiated royalty rate. Then,

$$\pi^A_{renegotiate} = p_A v - c_A + \frac{1}{1+\alpha_i}\left(b_i(r_i) - a(\mathbf{r})\right) = q_i(\mathbf{r})v + \widetilde{r_i}\alpha_i v \tag{31}$$

$$\pi^i_{renegotiate} = (1-p_i)\,\alpha_i v - c_i + \frac{\alpha_i}{1+\alpha_i}\left(b_i(r_i) - a(\mathbf{r})\right) = (1-\widetilde{r_i})\,\alpha_i v. \tag{32}$$

A's payoff is simply the sum of all the other royalty payments plus the renegotiated payment from B_i. B_i's payoff is simply its value $\alpha_i v$ of the innovation less its renegotiated royalty payment of $\widetilde{r_i}\alpha_i v$.

Solving equation 31 for $\widetilde{r_i}$ one can show that

$$q_i(\mathbf{r})v + \widetilde{r_i}\alpha_i v = p_A v - c_A + \frac{1}{1+\alpha_i}\left(b_i(r_i) - a(\mathbf{r})\right) \tag{33}$$

$$\widetilde{r_i} = \frac{1}{1+\alpha_i}\left(2\overline{p}_i + \frac{c_i/\alpha_i - c_A}{v} - q_i(\mathbf{r})\right) \tag{34}$$

where $\overline{p}_i = \frac{p_A+p_i}{2}$ (see appendix B.1 for the derivation). The same implicit license obtains from solving equation 32. Because $\widetilde{r_i}$ does not depend on r_i, strategic or myopic pre-trial bargaining is corrected by renegotiation. In other words, any initial r_i will be renegotiated to $\widetilde{r_i}$ if renegotiation occurs.[11]

The royalty rate from renegotiation makes intuitive sense. The term $q_i(\mathbf{r})$ represents the weighted average of all other negotiated royalties (for the moment those are assumed to be fixed). If this term is large, then the patent holder has a large opportunity cost of bringing suit against a particular licensee. Thus, B_i has more power to lower its payment r_i when others are willing to pay more (for this reason, we make q_i endogenous in the next section). Additionally, higher initial beliefs about

[11] Daughety and Reinganum (2004) point out that if bargaining occurs sequentially with multiple bargaining partners, then "most favored nation" clauses can be used strategically, and that this will affect the equilibrium contract price. In order to stay close to PK, we assume that bargaining with all licensees takes place simultaneously.

validity will raise the license fee, which is intuitive. Litigation costs can work in either direction; if c_i is large relative to c_A and v, then the payment to the patent holder will be higher (B_i's threat of going to court is less credible). Lastly, as α_i increases, it is presumed that its bargaining power increases, so $\widetilde{r_i}$ decreases.

Definition 1 *A royalty rate r_i^s is renegotiation-proof if, for any original r_i and bargaining rule, the implicit royalty resulting from renegotiation, $\widetilde{r_i}$, is equal to the renegotiation proof royalty, i.e., $\widetilde{r_i} = r_i^s$.*

The renegotiation-proof royalty ensures that the parties bargain optimally in the first instance, so that myopic and strategic bargaining are excluded. Strategic bargaining is excluded in the sense that an artificially high or low r_i cannot lead to a better outcome by forcing renegotiation, because the parties split the surplus at the time of the renegotiation.

Let $\overline{r_i}$ be defined as

$$\overline{r_i} = \overline{p_i} + \frac{c_i/\alpha_i - c_A}{2v}. \tag{35}$$

$\overline{r_i}$ represents a central tendency of the renegotiated royalty (as we show below). Equivalently, it is the renegotiated royalty if $\alpha_i = 1$ (so that there are no other licensees/infringers). With this definition, we can rewrite the renegotiation-proof license as

$$\widetilde{r_i} = \frac{1}{1+\alpha_i}\left(2\overline{p_i} + \frac{c_i/\alpha_i - c_A}{v} - q_i(\mathbf{r})\right) \tag{36}$$

$$\widetilde{r_i} = \frac{2\overline{r_i}}{1+\alpha_i} - \frac{q_i(\mathbf{r})}{1+\alpha_i} \tag{37}$$

This expression can be further simplified to be an implicit function of $\overline{r_i}$ and the average royalty.

$$\widetilde{r_i}\,(1+\alpha_i) = 2\bar{r}_i - q_i(\mathbf{r}) \tag{38}$$

$$\widetilde{r_i} = 2\overline{r_i} - \mathbf{a}'\mathbf{r} = \overline{r_i} + (\overline{r_i} - \mathbf{a}'\mathbf{r}) \tag{39}$$

The final expression for $\widetilde{r_i}$ provides another useful interpretation; the renegotiated royalty always approaches $\overline{r_i}$. The final royalty is $\overline{r_i}$ plus an adjustment that reflects how far $\overline{r_i}$ is from the weighted average of all licenses. If $\overline{r_i}$ tends to be large relative to the average license, then $\widetilde{r_i} > \overline{r_i} > \mathbf{a}'\mathbf{r}$. The larger is α_i, the more compressed the resulting differences will be.

3.4 Properties of the renegotiation-proof license

Equation 34 gives the renegotiation-proof royalty rate for a given set of "outside licenses," where outside refers to all licensees aside from i. Assuming these are fixed, we can show that the royalty rate has some attractive features. First, the renegotiation-proof royalty is unique.

Proposition 2 *For any bargaining rule and set of "outside" licenses q, there is a unique renegotiation-proof $\widetilde{r_i}$.*

Proof. For any positive range of cooperative surplus, there is a continuum of licenses that would lead to settlement. However, by definition, the particular bargaining rule will identify only one royalty from that continuum. Additionally, see appendix B.2. ■

Proposition 2 holds for a particular bargaining rule only. That is, each bargaining rule will yield a different renegotiation-proof royalty. Additionally, there may be other royalty rates that divide the surplus to settlement. However, for a particular bargaining rule, only $\widetilde{r_i}$ will divide the surplus for an arbitrarily small cooperative surplus.

Definition 3 *Consistent litigation preferences exist when both parties prefer to litigate or both parties prefer the status quo.*

Proposition 4 *The renegotiation-proof license will lead to litigation if and only if both parties prefer litigation, i.e., it makes litigation preferences consistent.*

Proof. See appendix B.3. ■

Hence, anytime there is a cooperative surplus, $\widetilde{r_i}$ avoids litigation without renegotiation. Conversely, whenever there is no cooperative surplus, $\widetilde{r_i}$ ensures that both parties prefer to litigate. In this way we can say that $\widetilde{r_i}$ makes litigation preferences "consistent." Considering the alternative, there are values for r_i that would lead one party to want to litigate and one party to want to settle. This situation is excluded in PK because the status quo is normalized to zero (since the parties may only choose between litigation and settlement). Their assumption holds when harm is exogenous, such as in some personal injury claims; however, even in these cases parties can generally choose precaution levels pre-harm.

In many instances, such as in patent cases (aside from unintentional infringement), the parties could have bargained prior to harm occurring. r_i indicates a hypothetical royalty rate that is proposed. The licensee/infringer decides whether to infringe and litigate or to accept the license. Should the patent holder intentionally choose a very high royalty, it will be inviting infringement. It is irrational to do so if there is a lower royalty that would be mutually agreeable. In that sense, the harm (infringement) is endogenous to the choice or royalty rate. In the model, an excessively high r_i would lead to renegotiation if there is a cooperative surplus. If there is not a cooperative surplus, then the proposed royalty rate is irrelevant to the final payoffs.

Similarly, an excessively low royalty is equally irrational. The licensee/infringer would accept the royalty only to have the patent holder renege and file suit. Again, renegotiation would yield an $\widetilde{r}_i$ which makes litigation preferences consistent, so that the parties agree to the license or agree to litigate.

The process of renegotiation is illustrated in figure 2, in the context of symmetric stakes.[12] In both panels, we graph the dollar value of the patent-holder's ask price and the license's bid price against different values of the royalty rate. Panel A reflects the situation where there is positive cooperative surplus to settling, and panel B reflects the litigation case. In both panels, royalty rates less than x will cause the patent holder to prefer litigation (its ask price will be positive). Similarly, high royalty rates (greater than y) will cause the licensee to prefer litigation. The renegotiation proof royalty is represented by z, which is always between x and y.

When there is no cooperative surplus (panel A) we should observe settlement. However, for very high levels of the royalty rate (above y in the figure) the licensee will prefer to litigate than to pay the royalty; i.e., B's bid is less than zero. Since A does not wish to litigate, it can attempt to split the cooperative surplus with B and move the royalty back into the settlement range. A similar situation happens when the royalty is very low (below x). In that case A prefers to litigate, and B will need to bribe A with some of the surplus in order to avoid litigation. Note that the cooperative surplus is positive for the entire range of royalty rates, but that litigation preferences of the two parties change.

In panel B of figure 2, we see the case where there is no cooperative surplus to settling. Here, values for the royalty below y will lead to a situation where A wants to litigate, but B does not. Similarly, royalties above x will make only B want to litigate. Renegotiation to $z = \widetilde{r}$ will cause both parties to want to litigate. Proposition 8 shows that the parties can not be made better off by considering inefficiently high or low royalties.

In the asymmetric stakes case, an inefficiently high royalty can cause the cooperative surplus to be positive even when it is negative at $\widetilde{r}_i$.[13] That is, it can appear that the parties can avoid litigation by choosing an arbitrarily high r. Proposition 8 demonstrates that this strategy can not make the parties better off than litigation.

Definition 5 *A party has efficient litigation preferences when it prefers to litigate if and only if there is no cooperative surplus.*

[12] The intuition holds for the asymmetric case.

[13] Note that the *Ask* and *Bid* lines are not parallel in the asymmetric stakes case.

Corollary 6 *The renegotiation-proof license will lead to litigation if and only if there is no cooperative surplus, i.e., it makes litigation preferences efficient.*

Corollary 6 follows from the fact that the renegotiation-proof license always splits the cooperative surplus (whether positive or negative). Because of this, both parties will always agree on litigation, and it will only occur if the cooperative surplus to bargaining is negative.

Corollary 7 *The renegotiation-proof license uniquely satisfies proposition 4 and corollary 6.*

As the cooperative surplus shrinks to a single point, the renegotiation-proof license is the only one that will avoid litigation, because it always splits the surplus. This is shown in the appendix B.2.

Proposition 8 *The renegotiation-proof license is Pareto efficient.*

Proof. See appendix B.4 ■

This proposition is crucial since it eliminates the possibility of choosing an "irrational" royalty in order to commit the parties to a better equilibrium. If the parties prefer litigation with $\widetilde{r_i}$, then litigation is efficient.[14]

3.5 Nash equilibrium licenses

The equilibrium $\mathbf{r}^*$ is a vector of royalty rates such that all parties bargain to a renegotiation proof license. Because $\widetilde{r_i}$ depends upon all the other licenses (in the $q_i(\mathbf{r})$ term), the equilibrium is a solution to a linear system of equations. Formally, let

$$\mathbf{r}^* = \begin{bmatrix} r_1^* \\ r_2^* \\ \vdots \\ r_N^* \end{bmatrix} \text{ and } \overline{\mathbf{r}} = \begin{matrix} \bar{r}_1 \\ \bar{r}_2 \\ \vdots \\ \bar{r}_N \end{matrix}$$

where $\mathbf{r}^*$ is a vector of renegotiation-proof royalty rates, and $\overline{r_i}$ is defined according to equation 35:

$$\overline{r_i} = \overline{p_i} + \frac{c_i/\alpha_i - c_A}{2v}. \tag{40}$$

From equation 39 the renegotiation-proof renegotiation proof royalty rate is implicitly defined as

$$\widetilde{r_i} = 2\overline{r_i} - \mathbf{a}'\mathbf{r}. \tag{41}$$

[14] Note that litigation is efficient only from the perspective of the parties involved and their divergent beliefs about the trial outcome.

Equilibrium occurs where each party negotiates to this royalty. The equilibrium is found by solving for all fees simultaneously. In the case of the equilibrium royalty $\mathbf{r}^*$ we can say that each individual royalty must satisfy

$$r_i^* = 2\overline{r_i} - \mathbf{a}'\mathbf{r}^*. \tag{42}$$

Or, in matrix notation

$$\mathbf{r}^* = 2\overline{\mathbf{r}} - \mathbf{i}\mathbf{a}'\mathbf{r}^* \tag{43}$$

where $\mathbf{i}$ is a column vector of ones.

Solving simultaneously,

$$\mathbf{r}^* + \mathbf{i}\mathbf{a}'\mathbf{r}^* = 2\overline{\mathbf{r}} \tag{44}$$

$$\left(\mathbf{I} + \mathbf{i}\mathbf{a}'\right)\mathbf{r}^* = 2\overline{\mathbf{r}} \tag{45}$$

$$\mathbf{r}^* = \left(\mathbf{I} + \mathbf{i}\mathbf{a}'\right)^{-1} 2\overline{\mathbf{r}} \tag{46}$$

The inverse of $\left(\mathbf{I} + \mathbf{i}\mathbf{a}'\right)$ is given by $\left(\mathbf{I} - \frac{1}{2}\mathbf{i}\mathbf{a}'\right)$, so

$$\mathbf{r}^* = \left(\mathbf{I} - \frac{1}{2}\mathbf{i}\mathbf{a}'\right) 2\overline{\mathbf{r}} \tag{47}$$

$$\mathbf{r}^* = 2\overline{\mathbf{r}} - \mathbf{i}\mathbf{a}'\overline{\mathbf{r}}. \tag{48}$$

For an individual license,

$$r_i^* = 2\overline{r_i} - \mathbf{a}'\overline{\mathbf{r}}. \tag{49}$$

So, an individual license can be expressed as a function of the difference between $\overline{r_i}$ and $\overline{\mathbf{r}}$.

Relating equation 49 with equation 42, one can see that $\mathbf{a}'\mathbf{r}^* = \mathbf{a}'\overline{\mathbf{r}}$ in equilibrium. This does not, of course, imply that $\mathbf{r}^* = \overline{\mathbf{r}}$, only that the weighted averages are equal. The intuition follows from the fact that $\widetilde{r_i} - \overline{r_i} = \overline{r_i} - \mathbf{a}'\mathbf{r}$. For a given set of licenses $\mathbf{r}$, if $\overline{r_i}$ differs from the weighted average of all licenses, then $\widetilde{r_i}$ is twice the distance from the weighted average. In equilibrium this is true for all licenses, so the individual differences from $\mathbf{a}'\overline{\mathbf{r}}$ negate one another, and $\mathbf{a}'\mathbf{r}^* = \mathbf{a}'\overline{\mathbf{r}}$. This value is, in fact, the patent holder's equilibrium revenue.

This fact is useful because $\overline{r_i}$ is dependent only on parameters identified with A and B_i. That is, it does not depend on the value of the other license. So, while we may not know each individual r_i^*, we do know that the weighted average royalty facing A is equivalent to $\mathbf{a}'\overline{\mathbf{r}}$.

Also, it is important to note that the solution holds for any arbitrary vector of market shares $\mathbf{a}$, meaning that the equilibrium allows for heterogeneity of firms. Of course, as N rises, the average α_i must decrease. Nonetheless, the solution accounts for the distribution of shares—or the concentration of licensees.

4 Litigation in Equilibrium

Proposition 9 *In equilibrium, all parties have consistent litigation preferences.*

Proof. From expression 21, in equilibrium litigation occurs between defendant i and the plaintiff if

$$p_A - \alpha_i p_i > q_i(\mathbf{r}^*) + \frac{c_A + c_i}{v}. \tag{50}$$

Substituting $q_i(\mathbf{r}^*) = \mathbf{a}'\mathbf{r}^* - \alpha_i r_i^*$, the condition becomes

$$p_A - \alpha_i p_i > (\mathbf{a}'\mathbf{r}^* - \alpha_i r_i^*) + \frac{c_A + c_i}{v}. \tag{51}$$

Using the fact that $\mathbf{a}'\mathbf{r}^* = \mathbf{a}'\overline{\mathbf{r}}$ and the definition of r_i^* from equation 49,

$$p_A - \alpha_i p_i > (\mathbf{a}'\overline{\mathbf{r}} - \alpha_i(2\overline{r}_i - \mathbf{a}'\overline{\mathbf{r}})) + \frac{c_A + c_i}{v}. \tag{52}$$

Finally, using the definition of $\overline{r_i}$ from equation 35,

$$p_A - \alpha_i p_i \quad > \quad (1+\alpha_i)\mathbf{a}'\overline{\mathbf{r}} - \alpha_i 2\left(\frac{p_A + p_i}{2} + \frac{c_i/\alpha_i - c_A}{2v}\right) + \frac{c_A + c_i}{v} \tag{53}$$

$$p_A(1+\alpha_i) \quad > \quad (1+\alpha_i)\mathbf{a}'\overline{\mathbf{r}} + (1+\alpha_i)\frac{c_A}{v} \tag{54}$$

The expression reduces to

$$p_A - \mathbf{a}'\overline{\mathbf{r}} > \frac{c_A}{v} \tag{55}$$

■

This result is very important: litigation between A and *any* licensee depends on the average license, in equilibrium. Thus, the condition is the *same for all licensees faced by the patent holder.* If the patent holder wishes to litigate against one licensee, then it will be willing to litigate against any licensee. The patent holder will, of course, only need to litigate against one licensee in order to obtain a decision from the court. It is important to note that this result follows from the fact that pre-dispute bargaining over the proposed license fee occurs in the shadow of the credible threats made by the respective parties. And the size of that shadow depends heavily on the precedential impact of the decision. Because the decision applies to all other related cases, the threat points depend on the market-wide pre-dispute bargaining by all players.

The intuition for expression 55 is that it is the average royalty that matters. If if it is too low relative to the patent holder's beliefs p_A, then the patent holder will want a decision from the court. If it is high, then patent holder will be unwilling to litigate against any particular low $\widetilde{r}_i$ licensee, because it risks losing the high average royalty if the patent is invalidated.

By expanding $\mathbf{a}'\overline{\mathbf{r}}$, we can write an expression for litigation that mimics that of PK.

We know that

$$\mathbf{a}'\overline{\mathbf{r}} = \sum_{i=1}^{N} \alpha_i \overline{r_i} \tag{56}$$

$$\mathbf{a}'\overline{\mathbf{r}} = \sum_{i=1}^{N} \alpha_i \frac{p_A + p_i}{2} + \sum_{i=1}^{N} \frac{c_i - \alpha_i c_A}{2v} \tag{57}$$

$$\mathbf{a}'\overline{\mathbf{r}} = \frac{p_A + \mathbf{a}'\mathbf{p}}{2} + \frac{\mathbf{i}'\mathbf{c} - c_A}{2v} \tag{58}$$

where $\mathbf{a}'\mathbf{p}$ is the weighted average of the licensees' beliefs and $\mathbf{i}'\mathbf{c}$ is the sum of all the licensees' litigation costs.

In equilibrium, if there is litigation we have shown above that the choice of defendant is irrelevant, since all parties have consistent litigation preferences. Thus c_i may be replaced by $\alpha_i c_i$ if B_i expects that it will bear the cost of litigation with probability α_i (e.g., a defendant is chosen at random based on market share). In that case equation 58 becomes

$$\mathbf{a}'\overline{\mathbf{r}} = \frac{p_A + \mathbf{a}'\mathbf{p}}{2} + \frac{\mathbf{a}'\mathbf{c} - c_A}{2v} \tag{59}$$

and the litigation condition can be written as

$$p_A - \mathbf{a}'\mathbf{p} > \frac{c_A + \mathbf{a}'\mathbf{c}}{v} \tag{60}$$

which is virtually identical to PK's result for *symmetric stakes*, with the exception that p_B is replaced by $\mathbf{a}'\mathbf{p}$ and c_B is replaced by $\mathbf{a}'\mathbf{c}$.

This result is important because the asymmetry of stakes does not affect the probability of litigation. That is, asymmetry does not substantially change the litigation rate—and consequently the observed win rate—relative to symmetric stakes. Showing that asymmetry is inconsequential would be, admittedly, a mundane result, were it not in contrast to the presumption of most of the literature on litigation selection and stakes.

While asymmetry does not directly impact the litigation condition, it does have an indirect impact. Asymmetry matters to the extent that the weighted average $\mathbf{a}'\mathbf{p}$ will have a lower variance that p_i, if licensees' draws of the facts of the case are independently distributed. In the PK model, lower variance leads to less litigation. Thus, relative to the symmetric stakes case, and assuming that each p_i is calculated on the basis of draws from the same distribution for all $i \in N$, then asymmetry will lead to *less* litigation.

Relative to the original model posed by PK, this model specifies both an upside and a downside to asymmetry in allowing an opportunity cost of litigation. PK assume that there is only an upside and that the downside is the status quo of zero. However, in practice, this is a special case and not likely to be true in the case of precedent.

The result of the model in terms of the litigation condition is quite intuitive: because the opportunity cost of litigation is endogenous, the upside and downside from asymmetry work in opposite directions and drive prices, and the litigation condition to the original PK result.

5 Discussion

The model presented in this paper brings the PK model "full circle" to allow for bargaining prior to harm. This *pre-dispute* bargaining creates an endogenous opportunity cost. The strategic bargaining by the parties in the shadow of precedent creates an equilibrium where the amount of harm caused by a failure to reach an agreement is endogenous. Because α_i drops out of the litigation condition, its only impact is from the distribution of beliefs and legal costs as they relate to α. There are two important implications: endogenous harm and selection effects.

Pre-dispute bargaining creates endogenous harm by the licensees/infringers, in that infringement on a higher r will lead to greater lost profits to the plaintiff than infringement on a lower r. The bargaining can be implicit or explicit, in the sense that rational infringing parties will recognize the lost opportunity to license at the going royalty rate when they infringe. In the model presented here, infringers presume that the royalty rate would be negotiated in the shadow of threats about litigation, the consequences of precedent, and beliefs about validity. Similarly, the patent holder recognizes that litigating against a small infringer will risk the royalty rates available from potential licensees who esteem the patent more highly.

The application is more broad than bargaining over a royalty: the ability to mitigate harm prior to a dispute goes well beyond the narrow application of patent infringement. In the context of torts, bargaining for a royalty rate equates to investment in precaution levels where precaution is bilateral. If harm can be mitigated by investment in precaution, then subsequent disputes, litigation, and court decisions will be subject to the incentives of parties to take precaution in the shadow of the law.

We also found that asymmetry does not directly impact the litigation rate (and hence the win rate). However, it does have consequences on bargaining. Consider a small infringer and a big infringer with shares $\alpha < \frac{1}{2}$ and $1 - \alpha > \frac{1}{2}$, respectively. In equilibrium, the bargaining represented by the large licensee's royalty gives the small firm *more* bargaining power: for a given set of beliefs it can bargain to a relatively low royalty rate because the patent holder is required to count the full cost of litigation. If it is content with the royalty from the large firm, the small firm can extract more surplus with a lower rate. However, if litigation costs are high, then the small firm's bargaining power is limited unless litigation costs are related to α.[15] If expected litigation costs are proportional to α, then in equilibrium the patent holder is indifferent to diffuse usage or concentrated usage, with the exception that diffuse ownership is less "risky," in terms of the distribution of p_i. If expected litigation costs are not proportional to α, then the patent holder can extract higher $\mathbf{a}'\mathbf{r}^*$.

Bargaining in the shadow of precedent impacts the interpretation of the reasonable royalty estimation of damages in patent litigation. What is the "reasonable royalty" to award if the patent holder prevails? Courts have determined that reasonable royalties be "based upon a hypothetical negotiation between the patent owner and the infringer,. . . with both parties to the negotiation assuming that the patent is valid and would be infringed but for the license" (Northlake v. Glaverbel, 72 F. Supp. 2d 893 [1999]). This decision explicitly acknowledges that prior bargaining over r is done in the shadow of precedent. Scholars have also suggested that, in the language of the model, the royalty will update to $r = 1$ (Sherry and Teece 2004).

With regard to the litigation rate, we find that even with asymmetric stakes litigation is essentially the same as in the original PK model. The important implication of this is on the selection effects of litigation. Because in equilibrium the litigation rate is essentially unchanged, then the observed win rate will be essentially unchanged relative to the PK model. So, we would not expect to see the win rate biased away from 50%, even in highly asymmetric situations. The empirical literature is mixed on this point. Waldfogel (1995) finds that win rates in patent cases are biased away from 50%, whereas Marco (2004) finds that the win rates of litigated cases are biased towards 50% relative to the population win rate. Additionally, if litigation preferences are consistent, then litigation could be modeled in a reduced form approach as a unilateral decision. Marco (2005) finds some evidence of this.

A few caveats should be mentioned with regard to the model. First, we have assumed that there is no signaling. Our model is one of divergent beliefs only, in order to remain close to the PK model. However, some scholars have found asymmetric information and signaling to be important (Meurer 1989, Choi 1998). However, as mentioned above, Daughety and Reinganum (1993) find that when the timing of the bargaining is endogenous, signalling does not occur. Alternatively, one could

[15] One way to have expected litigation costs proportional to α is to assume that in litigation in equilibrium each party expects to be sued with a probability proportional to its market share, as in expression 60.

assume that the beliefs the parties have about patent validity include the information contained about all other parties' beliefs.

Additionally, in equilibrium there is no observed infringement in our model: there is only licensing or immediate litigation and trial. One could relax the assumption of immediate litigation by including an enforcement cost or enforcement error, so that we could observe infringement "under the radar." While this would highlight some features of accommodated infringement, the results on asymmetric stakes are unlikely to change substantially.

Lastly, in extreme cases of very low α_i and very high c_i, it is possible for $r_i^* > 1$ without litigation. It is never the case that $\mathbf{a}'\mathbf{r}^* > 1$. If $r_i^* > 1$, it means that the licensee is receiving a negative net profit because $\pi_i = \alpha_i v - \alpha_i r v < 0$. This situation occurs because we implicitly assume that the licensees are committed to using the technology (whether by licensing or litigating). That is, we have advanced PK by one step by allowing for pre-dispute bargaining. However, we have not gone the further step to allow for users to opt out of the technology altogether. This extension could take two forms: (1) allow users to opt out whenever $r > 1$, and to take a payoff of zero; or, (2) endogenize α_i. Either extension would bring more realism to the model. However, the purpose in this paper is to compare the original PK model to a model of asymmetric stakes is the opportunity cost of litigation is endogenous. Taking further extensions are worthwhile, but would obscure the result here.

A last remark is that it is certainly possible that precedent set in one case could impact the bargaining by parties unrelated to the original litigants. An interesting research agenda would be to investigate the extent to which precedent is a public good, and to what extent individual litigants will pursue litigation when they do not experience the benefit (or cost) of that precedent.

A Figures

Figure 1: Priest and Klein simulation results

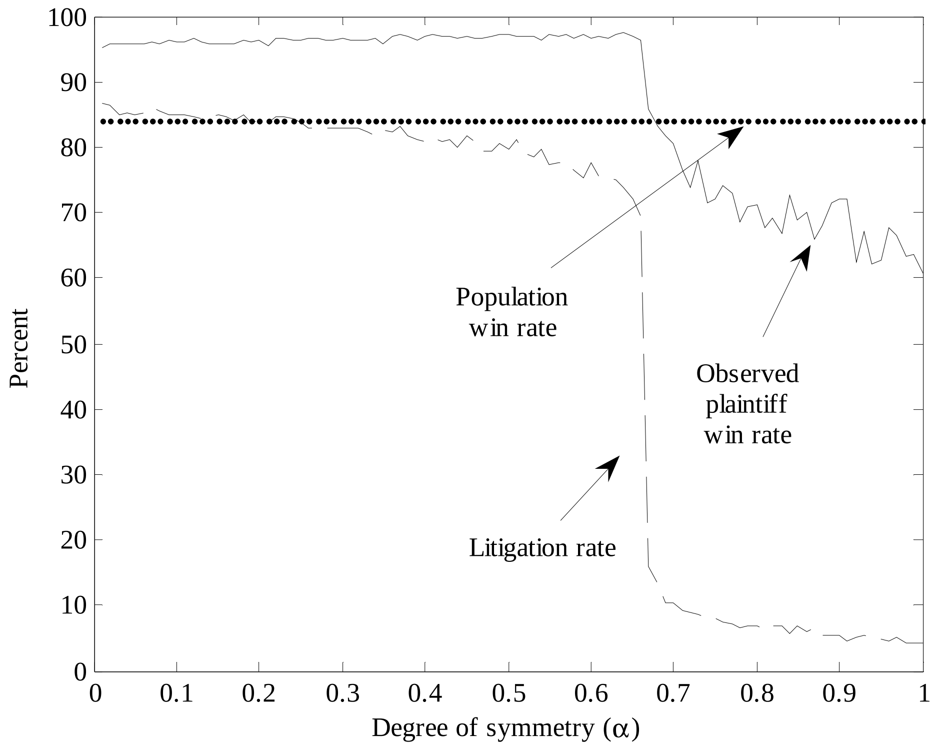
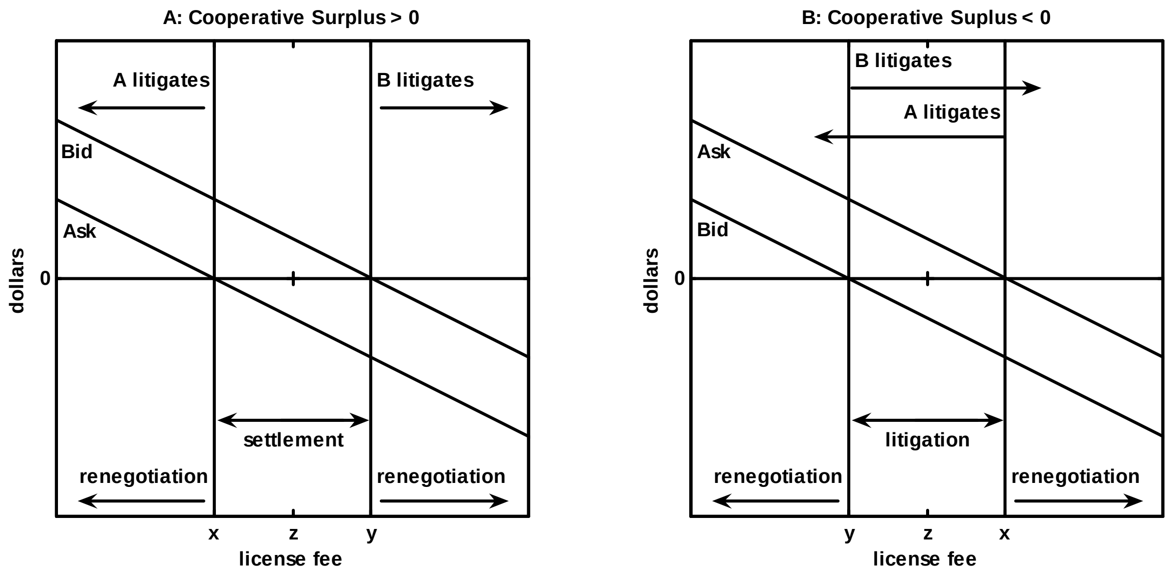


Figure 2: Renegotiation and litigation preferences



B Derivations

B.1 Solving for the implicit royalty

Solving equation 31 for $\widetilde{r}_i$ one can show that

$$\begin{aligned}
q_i(\mathbf{r})v + \widetilde{r}_i\alpha_i v &= p_A v - c_A + \frac{1}{1+\alpha_i}\left(b_i(r_i) - a(\mathbf{r})\right) \\
&= p_A v - c_A + \frac{1}{1+\alpha_i}\left((p_i\alpha_i v + c_i - r_i\alpha_i v) - (p_A v - c_A - r_i\alpha_i v - q_i(\mathbf{r})v)\right) \\
&= p_A v - c_A + \frac{1}{1+\alpha_i}\left(p_i\alpha_i v + c_i - p_A v + c_A + q_i(\mathbf{r})v\right) \\
\widetilde{r}_i\alpha_i v &= p_A v - c_A + \frac{1}{1+\alpha_i}\left(p_i\alpha_i v + c_i - p_A v + c_A + q_i(\mathbf{r})v\right) - q_i(\mathbf{r})v \\
&= (p_A v - c_A)\frac{\alpha_i}{1+\alpha_i} + \frac{1}{1+\alpha_i}\left(p_i\alpha_i v + c_i + q_i(\mathbf{r})v\right) - q_i(\mathbf{r})v \\
&= (p_A v - c_A)\frac{\alpha_i}{1+\alpha_i} + \frac{1}{1+\alpha_i}\left(p_i\alpha_i v + c_i\right) + \frac{1}{1+\alpha_i}q_i(\mathbf{r})v - q_i(\mathbf{r})v \\
\widetilde{r}_i &= \left(p_A - \frac{c_A}{v}\right)\frac{1}{1+\alpha_i} + \frac{1}{1+\alpha_i}\left(p_i + \frac{c_i}{\alpha_i v}\right) + \frac{q_i(\mathbf{r})/\alpha_i}{1+\alpha_i} - \frac{q_i(\mathbf{r})}{\alpha_i} \\
&= \left(p_A - \frac{c_A}{v}\right)\frac{1}{1+\alpha_i} + \frac{1}{1+\alpha_i}\left(p_i + \frac{c_i}{\alpha_i v}\right) + \frac{q_i(\mathbf{r})}{\alpha_i\left(1+\alpha_i\right)} - \frac{\left(1+\alpha_i\right)q_i(\mathbf{r})}{\left(1+\alpha_i\right)\alpha_i} \\
&= \left(p_A - \frac{c_A}{v}\right)\frac{1}{1+\alpha_i} + \frac{1}{1+\alpha_i}\left(p_i + \frac{c_i}{\alpha_i v}\right) - \frac{q_i(\mathbf{r})}{\left(1+\alpha_i\right)} \\
&= \frac{1}{1+\alpha_i}\left(p_A + p_i + \frac{c_i/\alpha_i - c_A}{v} - q_i(\mathbf{r})\right) \\
&= \frac{1}{1+\alpha_i}\left(2\overline{p}_i + \frac{c_i/\alpha_i - c_A}{v} - q_i(\mathbf{r})\right)
\end{aligned}$$

B.2 Proof of proposition 2: Uniqueness

Proposition 2 *For any bargaining rule and set of "outside" licenses q, there is a unique renegotiation-proof $\widetilde{r}_i$.*

Proof. Given other factors, any royalty that satisfies

$$\frac{p_A}{\alpha_i} - \frac{q_i(\mathbf{r})}{\alpha_i} - \frac{c_A}{\alpha_i v} \leq \widetilde{r}_i \leq p_i + \frac{c_i}{\alpha_i v}$$

will avoid litigation. Though when

$$\frac{p_A}{\alpha_i} - \frac{q_i(\mathbf{r})}{\alpha_i} - \frac{c_A}{\alpha_i v} = p_i + \frac{c_i}{\alpha_i v}$$

the only royalty that ensures

$$\frac{p_A}{\alpha_i} - \frac{q_i(\mathbf{r})}{\alpha_i} - \frac{c_A}{\alpha_i v} \leq \widetilde{r}_i \leq p_i + \frac{c_i}{\alpha_i v}$$

is the unique royalty

$$\frac{p_A}{\alpha_i} - \frac{q_i(\mathbf{r})}{\alpha_i} - \frac{c_A}{\alpha_i v} = \widetilde{r}_i = p_i + \frac{c_i}{\alpha_i v}$$

and since $\widetilde{r}_i$ always satisfies the condition, we see that $\widetilde{r}_i$ is the only royalty that always avoids litigation when cooperative surplus exists. ■

B.3 Proof of proposition 4: Consistent litigation preferences

Proposition 4 *The renegotiation-proof license will lead to litigation if and only if both parties prefer litigation, i.e., it makes litigation preferences consistent.*

Proof. Neither party will wish to litigate if

$$a(r_i, \mathbf{r}_{-i}) \leq 0 \leq b_i(r_i, \mathbf{r}_{-i}). \tag{61}$$

Royalties that satisfy this condition will allow parties to avoid litigation when litigation avoidance is possible. Conversely, when parties choose to litigate, no renegotiation will avoid litigation. The cooperative surplus is—after all—invariant to r_i in a dispute between B_i and A.

It suffices to show that if $b_i > a$, then $b_i(\widetilde{r}_i) \geq 0$ and $a(\widetilde{r}_i) \leq 0$. A royalty between B_i and A will avoid litigation without renegotiation if

$$\begin{aligned} a(r_i, \mathbf{r}_{-i}) &\leq 0 \leq b_i(r_i, \mathbf{r}_{-i}) \\ p_A v - \alpha_i r_i v - q_i(\mathbf{r}) v - c_A &\leq 0 \leq p_i \alpha_i v - \alpha_i r_i v + c_i \\ p_A v - q_i(\mathbf{r}) v - c_A &\leq \alpha_i r_i v \leq p_i \alpha_i v + c_i \\ p_A - q_i(\mathbf{r}) - \frac{c_A}{v} &\leq \alpha_i r_i \leq p_i \alpha_i + \frac{c_i}{v} \\ \frac{p_A}{\alpha_i} - \frac{q_i(\mathbf{r})}{\alpha_i} - \frac{c_A}{\alpha_i v} &\leq r_i \leq p_i + \frac{c_i}{\alpha_i v} \end{aligned} \tag{62}$$

Any royalty within the range defined by inequality 62 will avoid litigation when litigation is avoidable. Equivalently, as long as there is cooperative surplus, any royalty within the range of inequality 62 will cause both parties to prefer the status quo to litigation.

We now show that

$$\widetilde{r}_i \leq p_i + \frac{c_i}{\alpha_i v} \iff b_i - a \geq 0$$

Substituting the definition of $\widetilde{r}_i$ from equation 34,

$$\begin{aligned}
\frac{1}{1+\alpha_i}\left(2\overline{p}_i + \frac{c_i/\alpha_i - c_A}{v} - q_i(\mathbf{r})\right) &\leq p_i + \frac{c_i}{\alpha_i v} \\
p_A + p_i + \frac{c_i/\alpha_i - c_A}{v} - q_i(\mathbf{r}) &\leq (1+\alpha_i)\, p_i + (1+\alpha_i)\,\frac{c_i}{\alpha_i v} \\
p_A - \alpha_i p_i &\leq q_i(\mathbf{r}) + (1+\alpha_i)\,\frac{c_i}{\alpha_i v} - \frac{c_i/\alpha_i - c_A}{v} \\
p_A - \alpha_i p_i &\leq q_i(\mathbf{r}) + \frac{c_i + c_A}{v}.
\end{aligned}$$

The last inequality is the converse of expression 21, which is equivalent $b_i - a \geq 0$.

We now show that

$$\frac{p_A}{\alpha_i} - \frac{q_i(\mathbf{r})}{\alpha_i} - \frac{c_A}{\alpha_i v} \leq \widetilde{r}_i \Longleftrightarrow b_i - a \geq 0$$

Substituting the definition of $\widetilde{r}_i$ from equation 34,

$$\begin{aligned}
\frac{p_A}{\alpha_i} - \frac{q_i(\mathbf{r})}{\alpha_i} - \frac{c_A}{\alpha_i v} &\leq \frac{1}{1+\alpha_i}\left(2\overline{p}_i + \frac{c_i/\alpha_i - c_A}{v} - q_i(\mathbf{r})\right) \\
(1+\alpha_i)\,\frac{p_A}{\alpha_i} - (1+\alpha_i)\,\frac{q_i(\mathbf{r})}{\alpha_i} - (1+\alpha_i)\,\frac{c_A}{\alpha_i v} &\leq p_A + p_i + \frac{c_i/\alpha_i - c_A}{v} - q_i(\mathbf{r}) \\
(1+\alpha_i)\,\frac{p_A}{\alpha_i} - p_A - p_i &\leq \frac{c_i/\alpha_i - c_A}{v} - q_i(\mathbf{r}) + (1+\alpha_i)\,\frac{q_i(\mathbf{r})}{\alpha_i} + (1+\alpha_i)\,\frac{c_A}{\alpha_i v} \\
\frac{p_A}{\alpha_i} - p_i &\leq \frac{c_i/\alpha_i + c_A/\alpha_i}{v} + \frac{q_i(\mathbf{r})}{\alpha_i} \\
p_A - \alpha_i p_i &\leq q_i(\mathbf{r}) + \frac{c_i + c_A}{v}.
\end{aligned}$$

Again, the last inequality is the converse of expression 21, which is equivalent $b_i - a \geq 0$. ■

B.4 Proof of proposition 8: Pareto efficiency

Proposition 8 *The renegotiation-proof license is Pareto efficient.*

Proof. Cooperative Surplus Exists

Assume defendant i and the plaintiff initially face r_i^*. Now assume that cooperative surplus exists. Both the plaintiff and defendant prefer the existing situation to litigation, the conditions for settlement are met. The gross payoff to the plaintiff is

$$\pi_A = q_i(\mathbf{r})v + \alpha_i \widetilde{r}_i v$$

and the gross payoff to the defendant is

$$\pi_B = -\alpha_i \widetilde{r}_i v$$

Now suppose that the plaintiff and defendant change the license fee to r_i and that r_i is in the litigation-avoidance range. Gross Payoffs for the plaintiff and defendant become, respectively,

$$\begin{aligned} \pi_A &= q_i(\mathbf{r})v + \alpha_i r_i v \\ \pi_B &= -\alpha_i r_i v \end{aligned}$$

All else equal, this is clearly not a Pareto improvement. So when cooperative surplus exists, $\widetilde{r_i}$ is Pareto efficient.

No Cooperative Surplus

Assume defendant i and the plaintiff initially face $\widetilde{r_i}$. Now assume there is no cooperative surplus. We know from before that when $\widetilde{r_i}$ leads to litigation, all other license fees will also foster litigation. For any license fee, the respective plaintiff and defendant gross payoffs from litigation are

$$\begin{aligned} \pi_A &= A(\mathbf{r}){+}q_i(\mathbf{r})v + \alpha_i r_i v = p_A v - \alpha_i r_i v - q_i(\mathbf{r})v - c_A + q_i(\mathbf{r})v + \alpha_i r_i v = p_A v - c_A \\ \pi_B &= -B_i(\mathbf{r}) - \alpha_i r_i v = -p_i \alpha_i v + \alpha_i r_i v - c_i - \alpha_i r_i v = -p_i \alpha_i v - c_i \end{aligned}$$

and these payoffs are invariant to the license fee between defendant i and the plaintiff. Hence when there is no cooperative surplus, all else equal, $\widetilde{r_i}$ is still pareto efficient.

Regardless of whether $\widetilde{r_i}$ leads to litigation, no change in royalty is a pareto improvement. ■

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