Intergenerational Strategic Behavior and Crowding Out in a General Equilibrium Model

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Abstract

The return of large government budget deficits should encourage us to resume analysis of their effects. Two topics deserving further attention are the importance of correctly modeling the form of intergenerational relationships and clarification of the extent to which deficits crowd out private investment.

This paper presents an overlapping generations model in which children seek to manipulate the size of the end-of-life bequest they receive from the parent – similar to the manipulation observed in the Samaritan's dilemma. I first use numerical simulations to show this intergenerational strategic behavior does not negate the debt neutrality assertions of Ricardian equivalence.

Then, by introducing capital gains and inheritance taxes, I show the crowding out effect of government debt is notably smaller in models with strategic behavior; manipulation by children increases the importance of bequests, which forces parents to save (and bequeath) a larger portion of a debt-financed tax cut. In spite of the neutrality of debt under lump sum taxes, including intergenerational strategic behavior can significantly influence the outcome of government tax policies. Given the restrictive nature of the conditions required for Ricardian equivalence to hold, it may be more useful to measure how near to or far from Ricardian equivalence a particular policy or economy comes rather than simply determining whether or not it holds in that environment.

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1. INTRODUCTION

Interest in the effects of government budget deficits waned in the late 1990's as deficits in many countries substantially declined.[1] There is adequate empirical evidence that government debt *has* real effects (i.e., is not neutral,) and an assortment of suspected causes, but little agreement on the specific cause(s) of the failure of debt neutrality or, perhaps more importantly, on the significance of its failure.[2] The recent resurgence of significant fiscal deficits in many countries should motivate additional work on this topic.[3]

Previous research indicates a sizeable portion of wealth is accumulated out of a desire to leave intergenerational bequests; less clear is the exact motive for these bequests.[4] The impact of government deficits likely depends strongly on the exact nature of intergenerational relationships, a fact highlighted in Cremer and Pestieau (2003) and Kaplow (2001). I employ a little-studied version of intergenerational relationships to show that the effects of some fiscal policies can be quite sensitive to the nature of these relationships while other policies are neutral. Specifically, deficit financing is neutral when taxes are lump sum, but the introduction of capital gains and inheritance taxes reveals crowding out to be significantly smaller in versions of the model with strategic behavior than in one without strategic behavior. This suggests further work will be necessary to clarify the exact nature of intrafamily linkages in order to fully understand the impact of various fiscal policies.

Previous works present mixed results on the significance of intergenerational strategic behavior in determining the effect of fiscal policy changes. For example, Bruce and Waldman (1990) and Kotlikoff, Razin, and Rosenthal (1990) conclude government debt is not neutral in their respective static environments. Also, Seater (1993) writes that when strategic behavior is included in parent-child interactions "a debt-for-tax swap alters the threat point of the parents and/or the children and therefore has real effects, negating Ricardian equivalence." (p. 148). On the other hand, Bernheim and Bagwell (1988) conclude debt is neutral regardless of the nature of the intrafamily relationships. Seater, in his review of the Ricardian equivalence literature, finds only a handful of authors who attempt to make any connection between strategic behavior and Ricardian equivalence. One result of this paper is to demonstrate the intergenerational strategic behavior, by itself, does not necessarily cause Ricardian equivalence to fail.

General equilibrium analyses that include strategic behavior generally focus on other issues and don't evaluate Ricardian equivalence. The few exceptions include Nishiyama (2002), Gale and Perozek (2001), Smetters (1999), and Bernheim and Bagwell (1988). The model used by Nishiyama is similar to one of the model specifications used in this paper, but the analysis simply compares macro aggregates before and after a policy change (e.g., a ten percent tax cut). The analysis of Gale and Perozek employs the form of intergenerational strategic behavior studied here, but only does so in a partial-equilibrium framework. Smetters uses a general equilibrium framework to demonstrate the neutrality of some types of strategic behavior, but omits discussion of the form studied in this paper. Bernheim and Bagwell extend Barro's dynastic framework to extremes by asserting everyone is in fact connected through possible future marriages and through transfers to siblings, cousins, and charities (fn. 23). Their result that "everything is neutral" is certainly untenable (as they point out) and may stem in part from their assertion that uncertainty about potential future connections is irrelevant. However, the fact that government debt can be neutral under ideal circumstances tells us little about its effects under more common, distortionary, tax systems. This paper employs a specific type of intergenerational strategic behavior in order to assess its role in determining the effect of different fiscal policies. Specifically, I examine 1) whether Ricardian equivalence holds, and 2) the extent to which government debt 'crowds out' private investment when fiscal policy employs distortionary taxes.[5]

I model the interaction between an altruistic parent and a selfish child as a form of the Samaritan's dilemma. An altruistic parent makes an end-of-life transfer to a selfish child.[6] The child can attempt to elicit as large a transfer as possible from the parent by overconsuming when young, so as to be relatively poor when the parent is choosing the bequest amount.[7] The parent faces the problem of the good Samaritan: how to help the selfish individual without compromising his own consumption too much. Since successful manipulation by the child alters the margins at which decisions are made, the parent saves and transfers different amounts than he would without the strategic behavior. Consider the effect of a subsequent government substitution of debt for taxes in this environment. It seems unlikely the parent increases his transfer by the amount of a tax decrease when confronted with a manipulative child, causing

Ricardian equivalence to fail. I show that, in spite of the altered decision margins, Ricardian equivalence continues to hold in this framework. The intuition for this result relies on a revealed preference argument: In both cases the parent effectively controls allocation of the family's resources. The optimal allocation before a tax cut will still be optimal after the tax cut.

I examine three distinct specifications of this model. These specifications differ with respect to the timing of choices by parent and child within the model. Two specifications are considered manipulative since a parent conditions his bequest choice on his child's actions. A third specification is considered non-manipulative since a bequest amount is chosen at the beginning of an individual's life and is therefore independent of the child's actions. Results are obtained via computer simulation for each specification.

While incorporating intergenerational strategic behavior may not negate Ricardian equivalence, this tells us little (as noted above) about the effects of including strategic behavior under more realistic tax systems. Some authors suggest Ricardian equivalence may be most useful as a reference point – that we should measure how near to or far from Ricardian equivalence a particular policy or economy comes – rather than simply determining whether or not it holds.[8] The simulations offer us the opportunity to examine the extent to which government debt crowds out private investment when taxes are distortionary.[9] I next introduce several different combinations of inheritance, capital gains, and lump sum taxes and perform a series of Monte Carlo runs by varying the model parameters. I construct a measure of debt neutrality and assess the degree to which public debt reduces private investment in a variety of contexts. Appendix A offers an approximate calibration to the U.S. economy.

In all cases I find crowding out to be a significant concern. However, the portion of private investment crowded out by public debt is substantially smaller when strategic behavior is present than when it is absent. The cause of this difference is the fact that the strategic behavior leads families to place a greater importance on bequests than occurs when strategic behavior is absent. Therefore, saving by parents, particularly later in life, is also much more important when strategic behavior is present. This greater importance of saving in strategic model specifications causes consumers to save more of the proceeds of a debt-financed tax cut, thus producing smaller decreases in private investment (i.e., less crowding out), than occurs in the model specification without strategic behavior.

One of the important issues in this analysis, understanding the effects of changing inheritance and capital gains tax rates, is a topic of considerable current interest. One theory observes that parents who want to bequeath a specific amount to children will need to save more when the inheritance tax rate rises. If true, then reducing the inheritance tax rate should cause aggregate saving to decline. A competing theory holds that taxing bequests increases the cost of giving bequests, which leads people to shift resources away from giving bequests and away from saving for them. In this case, reducing the inheritance tax rate should cause aggregate saving to increase. Which of these theories holds in practice depends substantially on the precise motive for giving bequests, but remains an open question in economics.[10] The movement to reduce estate taxes in the U.S. will likely provide clearer evidence on this issue in the future. Cremer and Pestieau (2003) and Gale and Slemrod (2000) offer additional discussion of the relevant issues here.

In the dynastic model employed here, with altruism as the motive for bequests, it is expected that lower inheritance tax rates will increase bequests and aggregate saving.[11] This is consistent with, for example, the theoretical findings of Lainter (2001) and the empirical findings of Kopczuk and Slemrod (2001). Kopczuk and Slemrod also discuss the theoretical justifications for their result, examining the substitution and income effects at work when a government imposes an inheritance tax.

2. THE MODEL

The basic framework is a standard dynastic model with overlapping generations of three-period lived consumers. The use of three-period lived consumers provides both members of adjacent generations the opportunity to behave strategically. Only when the individuals can each make decisions in both of two overlapping periods can they make potentially manipulative choices in one period while still having a subsequent period of interaction. Consumers are homogeneous and there is no aggregate or individual uncertainty. Individuals are intertemporally linked by one-sided intergenerational altruism (parent to child). Each consumer has one child, born at the beginning of the parent's second period of life. In the third period of life a parent may transfer any nonnegative amount of resources to his child.[12]

The government finances production of a public good by levying taxes, issuing debt, or both. (I begin with only lump sum taxes in order to test Ricardian equivalence. Later, when examining crowding out, I add capital and inheritance taxes as well.) The amount of the public good and the financing method are exogenously specified. The public good enters consumers' utility function in an additively separable manner.

The economy has the following additional characteristics:

- A large finite number (N) of identical consumers is born at each time period.
- Each consumer is endowed with one unit of time in each period of life. This time is inelastically supplied as labor.
- A consumer born at time t may save (or borrow) an amount a_j^t at age j (j = 1,2). The net return on saving (cost of borrowing) initiated at time t is r_{t+1}. A consumer is not allowed to borrow against a possible future bequest he may receive, but may borrow against future wage income.
- The government collects per capita lump-sum taxes (τ_t), produces a public good (x_t), and can issue debt (D_t) in each period t. The government must eventually retire any debt it issues.
- The aggregate capital stock is the sum of private and public saving. That is,

$$K_t = Na_1^{t-1} + Na_2^{t-2} - D_t. \qquad (1)$$

- Consumers born at time t have preferences over their own consumption, their child's utility (U^{t+1}), and the public good as follows:

$$U^t \equiv ([u(c_1^t) + v(x_t)] + \beta[u(c_2^t) + v(x_{t+1})] + \beta^2[u(c_3^t) + v(x_{t+2})]) + \rho U^{t+1} \qquad (2)$$

  where $c_j^t (j = 1,2,3)$ is the age j consumption of a consumer born at time t. $\beta \in (0,1]$ is the intertemporal discount factor. $\rho \geq 0$ is the intergenerational discount rate.[13] Assume $u(\cdot)$ is strictly increasing and concave, $\lim_{c \to 0} u'(c) = \infty$ and $v(\cdot)$ is increasing.

- A single representative firm produces all goods for the economy according to the aggregate production function

$$Y_t = F(K_t, L_t)$$

where L_t is aggregate labor supplied at time *t*. F is strictly increasing and concave with respect to both arguments.

- Prices r_t and w_t are given by the time *t* marginal products of capital and labor respectively.

2.1. MANIPULATION IN AN OVERLAPPING GENERATIONS MODEL

The main issue here is specification of when the transfer amount is chosen. Two possibilities exist:

1. The parent chooses a bequest amount at the beginning of his life and is unable to deviate from that choice.[14]
2. The parent chooses the bequest amount in the final period of his lifetime.

To date, researchers using an overlapping generations model have consistently chosen some form of the first approach. We refer to this approach as one of 'precommitment' (to the future bequest amount) or as non-manipulative. The bequest amount is chosen during the parent's first period of life and cannot later be changed. While easier to compute, this approach introduces time consistency problems on the part of the parent. For example, a parent may wish to provide additional resources to a child who squandered resources when young or to give less to a child who saved a large amount when young, but is constrained from doing so.[15] The unrealistic nature of this restriction, combined with the time consistency problem, makes precommitment a difficult assumption to defend in practice.[16]

In the second approach the child's first period actions may influence the size of the bequest she receives. This gives both individuals the opportunity to behave strategically and is the primary focus of this paper. The strategy available to the child is to overconsume when young, in contrast to smoothing consumption over her lifetime. Later, when the parent is ready to choose a bequest amount, the child presents herself as a relatively poor individual and asks for a larger bequest. The child's ability to successfully manipulate the parent depends on the parent's affinity for the child and on both individuals' wealth and income levels. The child's interest in being manipulative depends primarily on her substitution rate between current and future consumption.

The parent may anticipate the potential for manipulation by his child. By slightly decreasing his second-period savings amount (which will still be much greater than the amount saved under precommitment) he can reduce the assets available to him when elderly. This diminishes the child's ability to elicit a larger bequest from the parent. The parent's success in mitigating the child's potential manipulation depends in part on the timing of their decisions within a period. One possibility is simultaneous choices of consumption, savings and bequests by all consumers alive in a period. A second possibility is sequential choices by the consumers alive in a period: oldest to youngest.[17] When the parent chooses his consumption and savings amounts first he is more successful at reducing the effect of the child's manipulation than he is when their choices are simultaneous.

I consider both of these specifications because it is not at all clear one is preferable to the other. The simultaneous choices approach is certainly more common but, as O'Connell and Zeldes (1993) points out, "In reality, of course, parents are born before children and make a large fraction of their consumption decisions before their children become independent adults. A more natural modeling approach would therefore be to make parents the 'leaders' in a sequential game."(p.364) Consideration of both specifications also helps demonstrate the robustness of the Ricardian equivalence and smaller crowding out results.

2.2. SIMULTANEOUS CHOICES

This section describes the model specification arising from the assumption of simultaneous consumption, savings, and bequest choices by the individuals alive in a period.

The standard way to analyze a representative-consumer economy with overlapping generations is to write out the consumer's objective function and all relevant constraints, differentiate with respect to all decision variables and construct the first-order conditions that govern the consumer's choices. One can readily take this traditional approach when a consumer's choices are independent of choices yet to be made by other individuals. This occurs when individuals can commit to an end-of-life bequest amount at the beginning of their life; that is, when the bequest amount depends only on the consumer's resources and his affinity for his child. However, the problem is more complicated when the choice of a bequest amount is dependent on choices yet to be made by other individuals – specifically, his child's first-period consumption (versus savings) choice. In this case the consumer must wait until the final period of his life (after the child's first period) to choose a bequest amount. Therefore, in describing the choices a consumer makes in this economy I begin with the problem facing a consumer in his last period of life and proceed using backwards induction.

The problem facing an elderly consumer (presented analytically below) requires analyzing the trade-off he experiences between using his resources for current consumption versus giving them to his child as a bequest. The fewer resources held by the child at this time, the greater will be the parent's bequest. The first order conditions governing this decision can be combined to form "decision rules" that specify how the elderly consumer should allocate his resources between consumption and bequest as a function of the resources he holds and of the resources his child holds. Working backwards, these decision rules will be used by the parent when making decisions in the preceding period (when middle-aged) about how many resources should be carried into the last period of life versus consumed in middle age. Similarly, the child will use knowledge of the decision rule governing the amount of bequest she will receive when making decisions in the preceding period (when young) about how much to consume or save that period.

The process continues by next considering the problem facing a middle-aged consumer. He faces a trade-off between current consumption versus saving for his elderly period. Saving more will allow him to consume more when elderly as well as allow him to give a larger bequest. The presence of strategic behavior means that the more he saves, the more his child will seek to obtain from him, thus providing a potential drawback to saving. Fortunately the "bequest rule," determined from analysis of the problem facing an elderly consumer, embodies all these considerations. Again, combining the consumer's first order conditions allows formulation of additional decision rules that govern middle-aged allocation of resources between current consumption and savings as functions of his savings decision when young and the bequest he receives. He will use knowledge of these rules when making decisions in the preceding period (when young) about how much to consume or save that period.

The backwards induction process concludes with analysis of the trade-off between current consumption versus saving that confronts a young consumer. Fortunately, the previously developed decision rules make clear how increasing savings will provide him more of his own resources in the future, but will lead to him receiving a lower bequest. Combining knowledge of these rules with his current-period budget constraint will allow him to choose the optimal consumption and savings amounts when young.

Perhaps this problem could be written down in a traditional manner, but doing so would require finding some way to capture the dependence of a consumer's bequest amount on his future child's first period consumption versus savings choice. We could certainly write the consumer's third period budget constraint as $c_3^t + B^t(a_1^{t+1}) = w_t + (1+r_{t+1})a_2^t$, where a_1^{t+1} is the child's first-period savings amount. However, since the child won't choose this amount until the second period of the parent's life, it isn't possible for the parent to solve this problem at the beginning of his life. Thus the backwards induction approach seems the most practical.

To better illustrate the interactions between members of different generations, in the descriptions below I take the perspective of examining the decisions by the three consumers alive within a particular period rather than tracking a single consumer across three time periods. The only real difference lies in the time superscripts. The resulting equilibrium is a type of Nash equilibrium.

Since the amount of the public good is exogenously specified and enters the utility function in an additively separable manner it has no effect on the consumption decisions of consumers. Therefore, for expositional clarity, I omit it from the descriptions below.

2.2.1. An Elderly Consumer

The consumer who is elderly at time t (born at time t – 2) chooses consumption, c_3^{t-2}, and a bequest, B^{t-2}, to maximize his utility from current consumption plus the discounted utility of his currently middle-aged child. Thus he solves

$$\max_{B^{t-2},c_3^{t-2}}[\beta^2 u(c_3^{t-2})+\rho[(\beta u(c_2^{t-1})+\beta^2 u(c_3^{t-1}))+\rho U^t]]$$

subject to the budget constraint facing an elderly consumer:[18]

$$c_3^{t-2}+B^{t-2}=w_t+a_2^{t-2}(1+r_t)-\tau_t, \tag{3}$$

the budget constraint currently facing his child:

$$c_2^{t-1}+a_2^{t-1}=w_t+a_1^{t-1}(1+r_t)+B^{t-2}-\tau_t$$

$$\text{and}\quad c_3^{t-2},B^{t-2}\geq 0$$

where $r_t, w_t, a_2^{t-2}, a_1^{t-1}$ and τ_t are taken as given. Also taken as given are the decision rules governing young and middle-aged consumer choices.

The primary concern of an elderly consumer is the trade-off between his own current consumption (c_3^{t-2}) and the impact his bequest will have on his child's current consumption (c_2^{t-1}) and savings (a_2^{t-1}). He knows the child faces a similar trade-off between *his* current consumption and savings. He also knows the child will use savings from this period to maximize utility next period. Assuming the child will act rationally in maximizing his utility means (by application of the envelope theorem) the child's current savings choice implicitly maximizes his current and future utility. Thus an elderly consumer need not explicitly incorporate his child's future decisions into his current maximization problem.

The elderly consumer's first order conditions can be combined to give

$$\rho u'(c_2^{t-1})-\beta u'(c_3^{t-2})=0. \tag{4}$$

Substituting in the respective budget constraints allows equation (4) to provide a specification of the parent's bequest as a function of the parent's second period savings amount and his child's first period savings amount. That is,[19]

$$B^{t-2}\equiv B(a_2^{t-2},a_1^{t-1}). \tag{5}$$

The child considers this bequest function when young while attempting to manipulate the size of her parent's bequest through her choice of consuming (c_1^{t-1}) versus saving (a_1^{t-1}).

2.2.2. A Middle-Aged Consumer

A consumer who is middle-aged at time t (born at time t–1) chooses consumption, c_2^{t-1}, and savings, a_2^{t-1}, to maximize his utility from present and future consumption plus the discounted utility (U^t) of his currently young child. Thus he solves the following problem:

$$\max_{a_2^{t-1}, c_2^{t-1}} [(\beta u(c_2^{t-1}) + \beta^2 u(c_3^{t-1})) + \rho U^t]$$

subject to the budget constraint facing a middle-aged consumer:

$$c_2^{t-1} + a_2^{t-1} = w_t + a_1^{t-1}(1 + r_t) + B^{t-2} - \tau_t, \tag{6}$$

the budget constraint he will face next period:

$$c_3^{t-1} + B^{t-1} = w_{t+1} + a_2^{t-1}(1 + r_{t+1}) - \tau_{t+1}, \tag{7}$$

his child's current budget constraint:

$$c_1^t + a_1^t = w_t - \tau_t$$

$$\text{and} \quad c_2^{t-1} \geq 0$$

where $r_t, w_t, a_1^{t-1}, B^{t-2}, a_1^t, c_1^t, \tau_t, w_{t+1}, r_{t+1}$ and τ_{t+1} are taken as given. Also taken as given are the decision rules governing young and elderly consumer choices.

Note that choosing a_2^{t-1} implicitly specifies B^{t-1} (from equation (5), taking a_1^t as given). Then equation (7) specifies c_3^{t-1} as a function of a_2^{t-1}. Assuming the child acts rationally, as will her child, etc., to maximize utility means (by application of the envelope theorem) the parent's current savings choice will be utility maximizing for all future generations as well as being optimal for him today. Thus a middle-aged consumer need not explicitly incorporate his child's future decisions into his current maximization problem.

We again combine the resulting first order conditions to develop implicit functions for a_2^{t-1} and c_2^{t-1} as functions of a_1^{t-1}, B^{t-2} and a_1^t.[20] The currently middle-aged consumer employed these functions when young (at time t – 1) to inform his choice of a_1^{t-1} and the current elderly consumer uses these functions to inform his choice of B^{t-2}. Also, the current young consumer uses awareness of these functions when choosing her current consumption and savings amounts.

2.2.3. A Young Consumer

A consumer who is young at time t chooses consumption, c_1^t and savings, a_1^t, to maximize her utility from present and future consumption plus the discounted utility of her as-yet-unborn child. Thus she solves the following problem:

$$\max_{c_1^t, a_1^t}[u(c_1^t) + \beta u(c_2^t) + \beta^2 u(c_3^t) + \rho U^{t+1}]$$

subject to the young consumer's current budget constraint:

$$c_1^t + a_1^t = w_t - \tau_t, \tag{8}$$

the budget constraint the young consumer will face next period:

$$c_2^t + a_2^t = w_{t+1} + a_1^t(1 + r_{t+1}) + B^{t-1} - \tau_{t+1} \tag{9}$$

$$\text{and} \quad c_1^t \geq 0$$

where w_t, a_2^{t-1} (used in equation (5) for B^{t-1}), τ_t, w_{t+1}, r_{t+1} and τ_{t+1} are taken as given. Also taken as given are the decision rules governing the choices of middle-aged and elderly consumers.

The young consumer will combine her knowledge of the decision rules developed for choices made by middle-aged and elderly consumers with knowledge about her parent's current savings (a_2^{t-1}) in order to manipulate the bequest her parent will choose next period. With these decision rules in place, we can see that the young consumer's savings choice implicitly specifies all future consumption and savings amounts. Thus by using backwards induction, and developing behavioral rules for each period of life, we've reduced the consumer's problem to one that can be solved in the first period of life.

The young consumer's first order conditions can be combined to show the trade-off between present consumption and savings:[21]

$$-u'(c_1^t) + \beta u'(c_2^t)[1 + r_{t+1} + \frac{\partial B^{t-1}}{\partial a_1^t}] = 0 \qquad (10)$$

where $\partial B^{t-1} / \partial a_1^t$ is obtained from equation (5).

2.3. SEQUENTIAL CHOICES

Uncertainty regarding the true nature of parent-child interactions, coupled with the fact that the sequential-choice specification produces different allocations than does the simultaneous-choice specification, strongly indicates we should evaluate it as well. However, since this specification is structurally identical to the simultaneous-choice specification I omit a detailed description of it here.

The primary difference between the two specifications is that here a middle-aged consumer does not treat the young consumer's savings choice as given. Analysis of the problem facing the young child allows us to formulate her savings choice as a function of the savings choice of today's middle-aged consumer. Recognizing the impact his savings choice will have on the child's savings choice increases the parent's ability to reduce the child's potential manipulative behavior.

3. COMPUTER SIMULATION

The specific functional forms used in the simulations are as follows.

- **Utility of Consumption** is given by

$$u(c) + v(x) = \frac{c^\gamma}{\gamma} + x \qquad (11)$$

  with $\gamma < 1$, $\gamma \neq 0$.

- **Production Function** is given by:

$$F(K_t, L_t) = AK_t^\alpha L_t^{(1-\alpha)} \qquad (12)$$

  where L_t is aggregate labor supplied at time *t* and $0 < \alpha < 1$.

- **Prices** are given by their respective marginal products so

$$r_t = A\alpha(L_t / K_t)^{(1-\alpha)} \quad \text{and} \quad w_t = A(1-\alpha)(K_t / L_t)^{\alpha}$$

3.1. AN EXAMPLE

To illustrate the differences between the equilibria of the different specifications I choose a set of parameter values (see Table 1) and compute the equilibrium of each specification. The values were chosen to be consistent with those used in other macroeconomic simulations. For example, a reasonable estimate of the annual intertemporal discount rate (β) is 0.96. Assuming each period in the model represents 20 years requires raising this value to the 20th power. The values of A and ρ were chosen to roughly scale the model economies and the flow of bequests to those observed in the U.S. over the last 20 years. Appendix A offers specific details regarding calibration of the different model specifications to observations of the U.S. economy. A significant difference between this example and the calibration discussed in Appendix A is that all government policy variables were set to zero for this example. The results are qualitatively insensitive to parameter variations.

Table 1: Parameters for Numerical Example

α	β	γ	ρ	A
0.3	0.442	-2.0	0.15	2.5

Table 2 compares the steady state equilibria of the three specifications. The first column shows the equilibrium resulting in the non-manipulative specification. Since individuals in this specification make all choices at the beginning of life, consumption is smoothed over all three periods. (That is, $u'(c_j) = \beta(1+r_{t+1})u'(c_{j+1})$, $j = 1,2$.) The parent chooses a bequest amount knowing his child will also smooth consumption over the three periods of her life. Thus the amount of bequest given depends only on the parent's affinity for his child and the *lifetime* wealth of each individual. As expected, in the non-manipulative specification, the parent gives the smallest bequest, first period consumption is smallest of any of the three specifications and second and third period consumption amounts are largest. Utility is lower under this specification, for similarly sized economies, than it is under the other two specifications.

The second column shows the equilibrium resulting in the simultaneous-choice specification. In this specification, since the return to saving is lower here than in the non-manipulative specification ($\partial B^{t-1}/\partial a_1^t < 0$ in equation (10)), consumers decrease first-period savings and increase first-period consumption. By having fewer assets when the parent chooses a bequest amount, children can successfully manipulate parents into giving significantly larger bequests. Compared to the other specifications, this specification produces the largest first period consumption amount, the smallest second and third period consumption amounts and the largest bequest amount. In addition, utility is the largest of any of the three specifications – primarily a result of shifting consumption forward in time.

Table 2: Steady State Comparison Across Specifications

	Non-Manipulative	Simultaneous Choices	Sequential Choices (A=2.5)	Sequential Choices (A=2.7162)
Period 1 Consumption	0.7022	0.8772	0.7351	0.8276
Period 2 Consumption	1.0068	0.9350	0.8486	0.9554
Period 3 Consumption	1.4434	1.3404	1.2166	1.3697
Period 1 Savings	0.0334	-0.1416	-0.0817	-0.0920
Period 2 Savings	0.1335	0.3085	0.1941	0.2185
Bequest Amount	0.1824	1.4519	1.0812	1.2173
Total Utility	-1.5046	-1.1259	-1.5273	-1.2049
Total Output	185,458	185,458	164,735	185,458

The child's decision to consume a large amount when young, and even to borrow to finance this consumption, increases the parent's marginal benefit of giving a bequest. This leads the parent to significantly increase his second period saving in order to finance the larger bequest. Note that roughly 70 percent of second-period saving will go to bequests in this specification, compared to roughly 20 percent in the non-manipulative specification. In addition, middle-aged individuals receive larger bequests than they did in non-manipulative specification. This additional income facilitates an even greater increase in saving and also allows him to increase second-period consumption, almost restoring it to the amount he enjoyed without a manipulative child.

In this specification, as in the non-manipulative specification, a middle-aged parent chooses his second-period savings amount taking the bequest of his elderly parent and the first-period savings of his child as given. Since these quantities are the inputs to the decision rules developed in the preceding section, the middle-aged consumer chooses to smooth consumption over his second and third periods of life.

Two columns are presented for the sequential-choice specification. The first uses the same parameter values as are used for the other two specifications. Note that the sequential-choice specification generates a lower output level than the other two specifications – the substantially lower second period savings produces a lower capital-labor ratio in this specification. This naturally causes lower consumption, savings and bequest amounts and a lower utility level. To better compare these amounts with those of the first two equilibria, multi-factor productivity is increased in the final column (to A = 2.7162) in order to bring total output up to the level observed in the first two specifications. Comparing the amounts in the final column to those of the simultaneous-choice specification reveals that under sequential choices a parent is able to somewhat mitigate his child's manipulation. This is evidenced by the lower bequest amount than that given under simultaneous choices. First period consumption falls slightly as first period savings rises. Second and third period consumption amounts rise slightly and second period savings falls substantially. The net result is a slightly lower utility level than that obtained in the simultaneous-choice specification. (Note, however, that utility in the unadjusted case, with A = 2.5, is even lower than that obtained in the non-manipulative specification.)

The parent can reduce his child's manipulation because he now makes his second-period choices *before* the child makes her first-period choices. By choosing first, and using the child's first-order condition to anticipate the savings amount the child will subsequently choose, the parent effectively chooses the bequest amount as well (using equation (5)). The parent chooses a smaller second-period savings amount (than in the simultaneous-choice specification) so as to have fewer extortable assets when elderly. The child reacts by choosing a larger first-period savings amount. With the child choosing after the parent, the child's choice will likely affect the amount of bequest the parent will give next period. As a result, individuals no longer smooth consumption over the second and third periods of life.

Comparisons to the non-manipulative specification follow the same pattern as described above for the simultaneous-choice specification. Also, the ratio of savings to output in the non-manipulative and simultaneous-choice specifications is 5.3 percent – similar to that observed in the U.S. over the last 20 years. This ratio falls to approximately 4.0 percent in the sequential-choice specification.

Perhaps the most significant inference to be drawn from this example is that, when strategic behavior is present, overconsumption by young consumers increases their dependence on bequests for the resources needed to finance consumption and saving in their second and third periods of life. This increased emphasis on bequests forces parents to save more in their second period of life and to spend a larger portion of their second-period saving on bequests. The increased importance of saving and bequests noted here for strategic model specifications will play a significant role in later assessment of the crowding out effect of government debt.

4. TESTING RICARDIAN EQUIVALENCE WITH LUMP SUM TAXES

Ricardian equivalence predicts that an altruistic parent will increase the size of his transfer to help his child (or other descendant) with their new tax burden. Using a wide variety of parameter values, I examine each specification and find that the substitution of government debt for lump sum taxes has no real effects. Savings and bequests increase temporarily as the proceeds of the tax cut are saved and passed on to future generations. This result is independent of the sequence of choices within a period and occurs in spite of the fact that altering the sequence of choices *does* change the resulting allocations.

To demonstrate the robustness of this result, simulation runs were made with a large number of different parameter configurations. For example, repayment of the debt was delayed several periods and different size deficits were considered. The results indicate Ricardian equivalence holds for all specifications.

As Seater (1993) observes, it is reasonable to expect Ricardian equivalence fails whenever a child successfully manipulates his parent's bequest amount. However, since this is not the result here we must re-evaluate our intuition. It is tempting to conclude the result is a natural outcome of the view that each family is actually "one big happy family" in which the parent effectively determines how the family's resources will be distributed. This analogy fails when we realize that the resulting allocations are not optimal, as evidenced by the fact that individuals achieve higher utility when allowed to behave strategically than when not.

Consider that each individual overlaps with a parent for two periods then, as the parent dies, becomes a parent himself and overlaps with his child for two periods, who subsequently overlaps with her child for two periods and so on. This effectively produces a sequence of two-period parent-child interactions. Rebelein (2002) shows that, in a static, two-period model with strategic behavior, shifting tax burdens from parent to child has no effect on consumption or savings. The question is, what happens when we introduce perturbations to the dynamic environment? Specifically, how do these perturbations propagate – do they expand or damp out? Given the subgame perfect nature of the equilibrium in each period small perturbations should damp out, rather than expand, over time. A small perturbation is defined as one that does not lead to corner solutions in a period. By construction we are studying only interior solutions so all perturbations are small by definition. Thus any change in the timing of taxes should damp out and Ricardian equivalence will hold.

More directly, Bernheim and Bagwell (1988) asserts that, when private transfers are operative, public transfers will be neutral when the game played between generations is strategically equivalent before and after the public transfer. Using their change-of-variable approach reveals that the substitution of debt for taxes produces a linear shift of the strategy space, but leaves the structure of the game unchanged (assuming transfers remain operative). This suggests we should expect Ricardian equivalence to hold in spite of the presence of strategic behavior. They also suggest their result to be independent of whether taxes are distortionary or lump sum, a conclusion different from that reached in the next section.[22]

5. DISTORTIONARY TAXES AND CROWDING OUT

When Ricardian equivalence holds private saving will increase by the amount of debt issued and private investment stays constant. When Ricardian equivalence fails private saving increases by some lesser amount (or may even fall) and private investment falls. The difference between the increase in private saving and the amount of debt issued equals the amount of private investment crowded out by the debt.

Given that Ricardian equivalence fails in the presence of distortionary taxes it may be useful to be able to gauge how close an approximation Ricardian equivalence is for a particular economy. The ratio of the amount of private investment crowded out to the amount of debt issued can be an effective measure of the impact of deficit financing on a particular economy.[23] The goal of this section is to determine whether the presence of intergenerational strategic behavior has any effect on the amount of crowding out caused by the substitution of government debt for current taxation and, if so, to clarify the nature of that effect.

5.1. ASSESSING CROWDING OUT

Let R be the degree to which Ricardian equivalence holds. I define R as

$$R = \frac{\Delta D - \Delta S}{\Delta D} \tag{13}$$

where

$$\Delta S = (a_1^t - \overline{a}_1 + a_2^{t-1} - \overline{a}_2)N \tag{14}$$

with $\overline{a}_j (j = 1, 2)$ being the steady state, pre-debt savings amounts and ΔD the increase in debt.

The value of R indicates the fraction of the debt that crowds out private investment and thus can be used to measure how far from satisfying Ricardian equivalence is the economy. When $R = 0$ the increase in private saving is equal to the increase in public debt and no crowding out occurs. If $R = 0.2$ in a particular environment then twenty percent of the debt crowds out private investment and we would assert there is a twenty percent deviation from Ricardian equivalence in that environment. The closer R is to zero the better Ricardian equivalence is as an approximation for that environment. The closer R is to one, the less Ricardian is the economy – individuals consume most of the proceeds of the tax cut rather than increasing saving to pay for future tax increases and private investment falls by an amount approaching the amount of the debt. Worth noting is that the decline in private investment will also negatively affect output, thereby further depressing aggregate saving.

Most often we have $0 < R < 1$, indicating consumers save only a portion of the tax cut and some crowding out occurs. It is possible to have $R > 1$, which could occur when the decrease in private investment combined with the tax increase necessary to pay interest on the debt lead to a decline in output (and bequests) that depresses private saving below its initial level. It is unclear what specific interpretation particular values of R have in these cases. At a minimum the fact that $R > 1$ indicates Ricardian equivalence fails dramatically; clearly, the government borrowing leads to a reduction in output, savings, etc., in these cases.

I assess the crowding out of private investment by debt when the debt is left outstanding indefinitely – a reasonable assumption given that governments today show little desire to repay the bulk of their outstanding debts. I model this by leaving debt outstanding for a length of time sufficient for the economy to reach a new steady state.[24] Leaving debt outstanding requires a small tax increase to pay the interest that accumulates each period. This experiment simulates the conditions countries with significant current public debts often experience.

5.2. DISTORTIONARY TAXES

For each specification I evaluate different combinations of lump sum, inheritance, and capital gains taxes. Let θ_B denote the tax rate on end-of-life bequests. Let θ_G denote the tax rate on capital gains. The new budget constraints for a consumer born in period t are as follows.

For a young consumer:

$$c_1^t + a_1^t = w_t - \tau_t. \tag{15}$$

For a middle-aged consumer:[25]

$$c_2^t + a_2^t = w_{t+1} + a_1^t(1+r_{t+1}) - \max(a_1^t, 0)r_{t+1}\theta_G + B^{t-1}(1-\theta_B) - \tau_{t+1}. \tag{16}$$

For an elderly consumer:

$$c_3^t + B^t = w_{t+2} + a_2^t(1+r_{t+2}) - \max(a_2^t, 0)r_{t+2}\theta_G - \tau_{t+2}. \tag{17}$$

The government's period t budget constraint is now

$$x_t + D_{t-1}(1+r_t) = 3N\tau_t + D_t + N\theta_B B^{t-2} + N\theta_G\left(\max(a_1^{t-1}, 0) + \max(a_2^{t-2}, 0)\right)r_t. \tag{18}$$

5.3. RESULTS AND DISCUSSION

To ensure generality of the results I performed a large number of runs; by varying the parameters of the model around the values given in Table 1, I constructed a grid of over 10,000 parameter combinations.[26] For each combination I determine the savings amounts for the initial, pre-debt, steady state and for the final, with-debt, steady state. I then calculate a value for *R* for each tax-policy combination for each model specification. The first column of Table 3 indicates the type of tax policy used to raise government revenues. The subsequent columns of Table 3 report average *R* values and standard deviations for each policy for the three different model specifications.

Most striking about the results is the significant difference between the *R* values obtained for the non-manipulative specification and those obtained for the two manipulative specifications. In contrast, the *R* values obtained for the two manipulative specifications are quite similar. Further, the standard deviations for the two manipulative specifications are consistently smaller than those of the non-manipulative specification. Also, note that there are qualitative similarities between the *R* values of the different model specifications; deviations from Ricardian equivalence are smallest when tax revenues are raised in some part with lump sum taxes, while deviations from Ricardian equivalence are largest when tax revenues are raised in some part with inheritance taxes.[27] Most of the *R* values are fairly large, indicating government debt crowds out a substantial amount of private investment when debt is left outstanding indefinitely.

Table 3: Average Portion of Debt Crowded Out of Private Investment

Policy[a]	Model Specification[b]		
	Non-Manipulative	Simultaneous Choices	Sequential Choices
Lump Sum Tax (Baseline)	0.0% (0.00)	0.0% (0.00)	0.0% (0.00)
Inheritance Tax Only	134.3% (58.0)	97.9% (31.9)	97.3% (35.3)
Capital Gains Tax Only	100.4% (21.4)	80.4% (13.8)	79.3% (15.7)
50% LS + 50% IN	60.4% (29.8)	43.4% (13.7)	43.0% (15.9)
50% LS + 50% CG	43.8% (10.3)	35.3% (6.40)	34.1% (7.10)
50% IN + 50% CG	114.5% (39.6)	88.9% (23.5)	88.5% (26.3)
1/3 LS + 1/3 IN + 1/3 CG	71.7% (27.3)	54.5% (13.8)	53.9% (15.9)

[a] LS refers to the lump sum tax; IN to the inheritance tax, and CG to the capital gains tax.
[b] Values in parentheses are standard deviations.

To understand why less crowding occurs in the manipulative specifications than occurs in the non-manipulative specification it may help to examine separately the effects of a temporary tax rate decrease and of a sustained increase in government borrowing.[28] For expositional clarity, the discussion below focuses on inheritance taxation; the results, however, are similar for capital gains taxation.[29]

In all three specifications, a one-period decline in the tax rate effectively provides individuals alive in that period with additional income. Because individuals care about the well-being of their child, they save and bequeath some of this additional income.[30] The next generation, while not benefiting directly from the tax cut, does receive additional income in the form of a larger bequest. These individuals also save and bequeath some of their additional income. Each successive generation also receives a larger bequest, and then gives a larger bequest, than they would have without the policy change. Since saving is an important part of setting aside resources for bequests, we expect aggregate saving to increase when bequests increase.[31]

An increase in government borrowing reduces the supply of funds available for private investment, which causes the interest rate to increase. The greater return to saving (higher cost of borrowing) causes individuals to save more (borrow less) throughout their lifetime. Then, in the final period of life, with more wealth than they would have had without the interest rate increase, individuals divide their additional wealth between purchasing more goods and giving a larger bequest. It is clear that aggregate saving will increase in this case and, since (at least) part of savings are used for bequests, that bequests should increase as well. The net result, in both the manipulative and non-manipulative model specifications, of a temporary tax cut and a long-lasting debt increase, is a persistent increase in aggregate saving and in bequests.

How does the presence of intergenerational strategic behavior influence these changes? It was noted earlier that manipulation by children forces parents to transfer, via bequests, more of their wealth than is required of parents with non-manipulative children. Thus, when parents receive additional income, whether explicitly, as from a tax cut or a larger bequest, or implicitly, as from an interest rate increase, we expect manipulated parents to save and bequeath a larger portion of additional income relative to the portion saved and bequeathed by unmanipulated parents. The greater increase in aggregate saving in the manipulative specifications means less crowding out occurs in these specifications than occurs in the non-manipulative specification. The results in Table 3 suggest crowding out in the manipulative specifications may be twenty to twenty-seven percent smaller than it is in the non-manipulative specification.

Finally, even though qualitative similarities exist between the effects occurring under inheritance and capital gains taxes, the results in Table 3 indicate that the deviation from Ricardian equivalence is consistently larger under an inheritance tax than it is under a capital gains tax. In other words, private saving increases more when there is a decrease in the capital gains tax rate than it does when there is a decrease in the inheritance tax rate. We know that decreasing either tax rate should cause saving to increase. However, a decrease in the capital gains tax rate *directly* increases the return to saving, while a decrease in the inheritance tax rate merely increases the return to one reason for saving: giving bequests. Since bequests are not the only motive for saving, it is not surprising that the increase in aggregate saving is more substantial when the capital gains tax rate declines than it is when the inheritance tax rate declines. This difference could be particularly noteworthy given the current policy debate in the United States regarding the future of the estate tax. The values in Table 3 suggest that using government debt to finance reductions in inheritance taxes will be more damaging to the economy – via greater crowding out and the accompanying reduction in output – than would using debt to finance reductions in capital gains taxes.

6. CONCLUSION

This paper has two primary goals. First, I extend the analysis of Ricardian equivalence and the intergenerational "Samaritan's dilemma" to a dynamic, general equilibrium environment by constructing a model with overlapping generations of three-period lived consumers. Analyzing computer simulations of this model shows the temporary substitution of government debt for current lump sum taxes has no effect on consumption, aggregate savings, or output in this framework. This result is independent of the sequence of choices undertaken within a period and of the specific parameter values selected.

The more substantial goal of this paper is to compare the crowding out effect of government debt in model specifications with and without intergenerational strategic behavior when taxes are distortionary. The amount of crowding out is much greater in the specification without strategic behavior than it is in the specifications with strategic behavior. The exact form of strategic behavior (simultaneous vs. sequential choices) has little impact on the results. In the specifications with strategic behavior the increased significance of bequests – the child's overconsumption when young forces greater reliance on bequests for income later in life – means parents give a larger bequest than they do in the specification without strategic behavior. Giving a larger bequest requires a parent to save more in the strategic specifications. In particular, when offered additional current income (e.g., via a tax cut), a manipulated parent is forced to bequeath and save a larger portion of the new income compared to the portion bequeathed and saved by an unmanipulated parent. This larger increase in saving when strategic behavior is present leads to a smaller amount of crowding out in these specifications.

The neutrality of lump sum taxes and smaller amount of crowding out with distortionary taxes might suggest further studies of the effects of deficit financing need not include strategic behavior. However, allowing strategic behavior does change the resulting allocations. We also note that non-strategic specifications can suffer time consistency problems and will overestimate the amount of crowding out occurring in manipulative families. The bottom line here is that the exact nature of intergenerational relationships affects the outcome of fiscal policy in a significant way. Thus it seems important to intensify our efforts to determine the true nature of these relationships.

APPENDIX A

A.1. CALIBRATION TO THE U.S. ECONOMY

The model is roughly calibrated to the U.S. economy as described below. The time period for observation is 1985-2004, chosen for its high value of new federal debt. (Twenty years equals one period in the model.)

- The average U.S. population of individuals 20+ years of age between 1985 and 2004 was 188,021 thousand.[32] Assuming 1/3 of these individuals belong to each age cohort gives

$$N = 62{,}673.7 \text{ thousand.}$$

- Total Federal Outlays were \$33,034.46 billion (in chained 2000 dollars) and Transfer Payments were \$17,802.07 billion during the period.[33] The difference of \$15,232.39 is considered to be the amount of the public good provided by the government. Thus

$$x_t = 15{,}232.39 \quad \forall\, t = 1, \ldots, \infty.$$

- The U.S. government issued \$3,301.34 billion (in chained 2000 dollars) of new federal debt during the period.[34] I assume this debt is issued in period T_D and, for simplicity, no debt is issued in other periods. Thus

$$D_t = \begin{cases} 0 & \forall\ t = 1, \ldots, T_D - 1, T_D + 1, \ldots, \infty, \\ 3{,}301.34 & \text{for } t = T_D. \end{cases}$$

- Assuming the tax burden is distributed evenly across all individuals alive at time t, the required per capita lump sum tax rate (without debt) is 15,232.39/188,021 = 0.08101 per period.[35] At time $t = T_D$, the government decreases the tax rate and instead borrows to finance some of its expenditures.

- Assuming the benefits of the tax cut are shared equally, the per capita amount of the tax cut is $\Delta\tau = 3{,}301.34 / 188{,}021 = 0.01756$.[36] The debt and any accumulated interest are repaid in the following period. Thus

$$\tau_t = \begin{cases} 0.08101 & t = 1, \ldots, T_D - 1, T_D + 2, \ldots, \infty, \\ 0.08101 - \Delta\tau & t = T_D, \\ 0.08101 + \Delta\tau(1 + r_{T_D+1}) & t = T_D + 1. \end{cases}$$

A.2. THE INTERGENERATIONAL DISCOUNT RATE

Reliable estimates of ρ are difficult to find in the literature. Some authors perform econometric analyses to test for the *presence* of intergenerational altruism but, even when they find evidence of it, seldom report the resulting intergenerational discount rate.[37] This absence led Laitner (1993) to simply assume parents care as much about their children's well-being as they do about their own and set $\rho = \beta$ in his analysis. One estimate is provided by Kuehlwein (1993), using the Retirement History Survey data. He estimates the ratio of the marginal utility of giving a bequest to the marginal utility of future consumption to be 1.2. That is, individuals choose bequests so the marginal utility of giving the bequest is 20 percent greater than the discounted marginal utility of consumption, which translates to an intergenerational discount rate of $\rho = \beta/1.2 = 0.368$.

The source of parent-to-child bequest data I use for calibration of the intergenerational discount rate is the Panel Study of Income Dynamics (PSID). The PSID has collected information on inheritances received since 1988. While data from recent years is not yet fully available, there is data for the years 1988 - 1993. During that time there are 612 instances of inheritances received by individuals. These inheritances range from under \$10 to over \$1,000,000 (1 case.) After adjusting for inflation (to constant 2000 dollars) the average inheritance received *per capita* is \$658.5, which translates to a per capita bequest of 0.01317 each period of the model.[38] The average inheritance amount, when one is given, is \$30,202.

Extrapolating from the PSID data gives an average annual flow of parent-to-child bequests of over \$50 billion. This is not out of line with Avery and Rendall's (1994) estimate of \$39.4 billion – which they observe is likely an underestimate.

Table A1 gives calibration results for the different model specifications.[39] The calibrated values give the economy a steady state GDP consistent with that of the U.S. over the last 20 years (one period in the model).[40] Converting the per-20-year-period intergenerational discount rates from

Table A1 to annual discount rates gives 0.9020, 0.8785 and 0.8859 respectively for the three model specifications.

Table A1: Calibration Values for A and ρ

Model Specification	Parameter Value	
	A	ρ
Non-Manipulative	2.3246	0.1272
Simultaneous Choices	2.7710	0.0750
Sequential Choices	2.8377	0.0886

A.3. OTHER PARAMETERS

Capital's share of income (α) is chosen to be consistent with values used in other similar studies. Kydland and Prescott (1988) calculate capital's factor share for 1977, "a typical year" (p. 352), to be 36 percent. Auerbach, Kotlikoff, Hagemann and Nicoletti (1989), in extending the overlapping generations model of Auerbach and Kotlikoff (1987) to include bequests, use a capital income share of 0.25. For this study I choose an intermediate value of $\alpha = 0.3$.

An appropriate value for the CES utility parameter γ is not readily obtainable from the data. Its value is chosen to be consistent with that used in related studies. Kydland and Prescott (1988) use a value of -0.5 while Auerbach, Kotlikoff, Hagemann and Nicoletti (1989) use a value of -1.86 and Auerbach and Kotlikoff (1987) use –3. Since the model of Auerbach, Kotlikoff, Hagemann and Nicoletti is most similar to my model I choose a value of –2. (The simulation results are relatively insensitive to the choice of γ.) This corresponds to an intertemporal elasticity of substitution of 0.33.

Appendix B

Derivation of First Order Conditions

This appendix presents derivations of equations used in the computer simulations. These equations are derived for the sequential choices model specification. Comments indicate how the equations would need to be modified for the simultaneous choices and precommitment specifications.

B.1 The Elderly Consumer

The elderly consumer at time t (born at time $t-2$) chooses consumption, $c_3^{t-2} \in \mathcal{R}_+$, and a bequest, $B^{t-2} \in \mathcal{R}_+$, to solve the following problem:

$$\max_{B^{t-2}, c_3^{t-2}} \left[\beta^2 u(c_3^{t-2}) + \rho\Big[\Big(\beta u(c_2^{t-1}) + \beta^2 u(c_3^{t-1})\Big) + \rho U^t\Big]\right]$$

subject to

$$c_3^{t-2} + B^{t-2} \leq w_t + a_2^{t-2}(1+r_t) - \tau_t$$

$$c_2^{t-1} + a_2^{t-1} \leq w_t + a_1^{t-1}(1+r_t) + B^{t-2} - \tau_t$$

$$\text{and } c_3^{t-2}, B^{t-2} \geq 0$$

where $r_t, w_t, a_2^{t-2}, a_1^{t-1}$, and τ_t are taken as given. (In the simultaneous choice and precommitment specifications c_2^{t-1} and a_2^{t-1} are also taken as given.)

The elderly consumer's first order conditions are most easily evaluated by substituting the budget constraints into his objective function, thereby eliminating the consumption terms, and differentiating with respect to B^{t-2}. This obtains:

$$-\beta^2 u'(c_3^{t-2}) + \rho\beta u'(c_2^{t-1})\frac{\partial c_2^{t-1}}{\partial B^{t-2}} + \rho\beta^2 u'(c_3^{t-1})\frac{\partial c_3^{t-1}}{\partial B^{t-2}} + \rho^2\frac{\partial U^t}{\partial B^{t-2}} \leq 0.$$

We next want to determine $\frac{\partial c_2^{t-1}}{\partial B^{t-2}}$, $\frac{\partial c_3^{t-1}}{\partial B^{t-2}}$ and $\frac{\partial U^t}{\partial B^{t-2}}$. One possibility is to consider the effect of B^{t-2} on each savings term of the consumer born at time $t-1$, as well as the effect on *his* bequest choice. Similarly we would need to consider the effect of B^{t-2} on the decisions of the consumer born at time t and so on. Much easier is to apply the envelope theorem, realizing these individuals maximize their choices subject to the choice of B^{t-2}. Then $\frac{\partial c_2^{t-1}}{\partial B^{t-2}} = 1$ and $\frac{\partial c_3^{t-1}}{\partial B^{t-2}} = \frac{\partial U^t}{\partial B^{t-2}} = 0$. This result is consistent with that of other authors (e.g., Altig and Davis (1989), O'Connell and Zeldes (1993), and Caballé (1995)).

The elderly consumer's first order condition now reduces to

$$\rho u'(c_2^{t-1}) - \beta u'(c_3^{t-2}) \leq 0. \tag{B.1}$$

Substituting in the respective budget constraints, using the CES utility functions, and rearranging gives

$$B^{t-2} \geq \frac{\rho^\sigma(w_t + a_2^{t-2}(1+r_t) - \tau_t) - \beta^\sigma(w_t - a_2^{t-1} + a_1^{t-1}(1+r_t) - \tau_t)}{\rho^\sigma + \beta^\sigma} \tag{B.2}$$

where $\sigma = \frac{1}{1-\gamma}$.

Since consumers are identical, elderly consumers in each time period face a similar bequest function – different only in the time sub- and superscripts.

The young consumer may use his parent's bequest function when attempting to manipulate the size of his parent's bequest.

B.2 The Young Consumer

The young consumer, born at time t, chooses consumption, $c_1^t \in \mathcal{R}_+$ and savings, $a_1^t \in \mathcal{R}$, to solve the following problem:

$$\max_{c_1^t, a_1^t} \left[u(c_1^t) + \beta u(c_2^t) + \beta^2 u(c_3^t) + \rho U^{t+1} \right]$$

subject to

$$c_1^t + a_1^t \leq w_t - \tau_t$$

$$c_2^t + a_2^t \leq w_{t+1} + a_1^t(1+r_{t+1}) + B^{t-1} - \tau_{t+1}$$

$$\text{and } c_1^t \geq 0$$

where $w_t, a_2^{t-1}, \tau_t, w_{t+1}, r_{t+1}, \tau_{t+1}$ and equation (B.2) are taken as given. (In precommitment the young consumer also takes B^{t-1} as given.)

The young consumer may use information about his parent's resources (a_2^{t-1}) to attempt to manipulate the bequest his parent will choose next period – hence the inclusion of equation (B.2) as a constraint.

The young consumer's first order order conditions are most easily determined by substituting the budget constraints into his objective function, thereby eliminating the consumption amounts, and differentiating with respect to a_1^t. Applications of the envelope theorem give $\frac{\partial c_3^t}{\partial a_1^t} = 0$ and $\frac{\partial U^{t+1}}{\partial a_1^t} = 0$.

We then obtain the following result (equation (10) in the text):

$$-u'(c_1^t) + \beta u'(c_2^t)\left[1 + r_{t+1} + \frac{\partial B^{t-1}}{\partial a_1^t}\right] \leq 0.$$

Evaluate $\frac{\partial B^{t-1}}{\partial a_1^t}$ using equation (B.2), which gives

$$\frac{\partial B^{t-1}}{\partial a_1^t} = \frac{-\beta^\sigma(1 + r_{t+1})}{\rho^\sigma + \beta^\sigma}.$$

Using this result and the CES utility functions, we substitute in the appropriate budget constraints and solve for a_1^t. This gives

$$a_1^t \geq \frac{(w_t - \tau_t)A_t + a_2^t - B^{t-1} - w_{t+1} + \tau_{t+1}}{A_t + 1 + r_{t+1}} \tag{B.3}$$

where

$$A_t = \left[\frac{\beta(1 + r_{t+1})\rho^\sigma}{\rho^\sigma + \beta^\sigma}\right]^\sigma.$$

Note: For the precommitment specification a young consumer's savings choice has no effect on the parent's bequest choice. Thus $\frac{\partial B^{t-1}}{\partial a_1^t} = 0$ and

$$A_t = \left[(1 + r_{t+1})\beta\right]^\sigma.$$

Since consumers are identical, young consumers in each time period face a similar first period savings function – different only in the time sub- and superscripts. The middle-aged consumer may use his child's first period savings function when seeking to minimize the child's manipulative behavior.

B.3 The Middle-Aged Consumer

The middle-aged consumer at time t (born at time $t-1$) chooses consumption, $c_2^{t-1} \in \mathcal{R}_+$, and savings, $a_2^{t-1} \in \mathcal{R}$, to solve the following problem:

$$\max_{a_2^{t-1}, c_2^{t-1}} \left[\left(\beta u(c_2^{t-1}) + \beta^2 u(c_3^{t-1}) \right) + \rho U^t \right]$$

subject to

$$c_2^{t-1} + a_2^{t-1} \leq w_t + a_1^{t-1}(1 + r_t) + B^{t-2} - \tau_t$$

$$c_3^{t-1} + B^{t-1} \leq w_{t+1} + a_2^{t-1}(1 + r_{t+1}) - \tau_{t+1}$$

$$c_1^t + a_1^t \leq w_t - \tau_t$$

and

$$c_2^{t-1} \geq 0$$

where $r_t, w_t, a_1^{t-1}, B^{t-2}, \tau_t, r_{t+1}, w_{t+1}, \tau_{t+1}$, and equations (B.2) and (B.3) are taken as given. (In the simultaneous-choice and precommitment specifications c_1^t and a_1^t are also taken as given.)

The middle-aged consumer's first order conditions are most easily determined by substituting the budget constraints into his objective function, eliminating the consumption terms, and differentiating with respect to a_2^{t-1}. Remember, in the sequential choices specification the choice of a_2^{t-1} affects a_1^t and the choice of a_1^t affects B^{t-1}.

We obtain the following first order condition:

$$-\beta u'(c_2^{t-1}) + \beta^2 u'(c_3^{t-1}) \left[1 + r_{t+1} - \frac{\partial B^{t-1}}{\partial a_2^{t-1}} - \frac{\partial B^{t-1}}{\partial a_1^t} \frac{\partial a_1^t}{\partial a_2^{t-1}} \right] + \rho \frac{\partial U^t}{\partial a_2^{t-1}} \leq 0. \tag{B.4}$$

Evaluation of the last term gives

$$\frac{\partial U^t}{\partial a_2^{t-1}} = u'(c_1^t)\Big[-\frac{\partial a_1^t}{\partial a_2^{t-1}}\Big] + \beta u'(c_2^t)\Big[(1+r_{t+1})\frac{\partial a_1^t}{\partial a_2^{t-1}} + \frac{\partial B^{t-1}}{\partial a_2^{t-1}} + \frac{\partial B^{t-1}}{\partial a_1^t}\frac{\partial a_1^t}{\partial a_2^{t-1}}\Big].$$

Consider the four terms on the right-hand side of this equation. The first, second and fourth terms together equal zero by application of the envelope theorem (from the young consumer's first order condition.)

So equation (B.4) becomes

$$-\beta u'(c_2^{t-1}) + \beta^2 u'(c_3^{t-1})\Big[(1+r_{t+1}) - \frac{\partial B^{t-1}}{\partial a_2^{t-1}} - \frac{\partial B^{t-1}}{\partial a_1^t}\frac{\partial a_1^t}{\partial a_2^{t-1}}\Big] + \rho\beta u'(c_2^t)\frac{\partial B^{t-1}}{\partial a_2^{t-1}} \leq 0. \quad \text{(B.5)}$$

Use the elderly consumer's first order condition to apply the envelope theorem again. This eliminates the third and fifth terms (of the five terms) of equation (B.5).

Now evaluate $\frac{\partial B^{t-1}}{\partial a_1^t}$ and $\frac{\partial a_1^t}{\partial a_2^{t-1}}$.

Note: a_1^t is not directly affected by a_2^{t-1}; instead the effect is through the parent's bequest choice B^{t-1}. So

$$\frac{\partial a_1^t}{\partial a_2^{t-1}} = \frac{\partial a_1^t}{\partial B^{t-1}}\frac{\partial B^{t-1}}{\partial a_2^{t-1}}.$$

Evaluate $\frac{\partial B^{t-1}}{\partial a_1^t}$ and $\frac{\partial B^{t-1}}{\partial a_2^{t-1}}$ using equation (B.2) and evaluate $\frac{\partial a_1^t}{\partial B^{t-1}}$ using equation (B.3).

Now equation (B.5) reduces to

$$-u'(c_2^{t-1}) + \beta u'(c_3^{t-1})F_t \leq 0 \quad \text{(B.6)}$$

where

$$F_t = 1 + r_{t+1} - \frac{\rho^\sigma \beta^\sigma (1+r_{t+1})^2}{(\rho^\sigma + \beta^\sigma)^2 (A_t + 1 + r_{t+1})}.$$

Note: For the simultaneous choices and precommitment specifications $\frac{\partial a_1^t}{\partial a_2^{t-1}} = 0$. (In these specifications the middle-aged consumer takes his child's current-period savings amount (a_1^t) as given when choosing a savings amount a_2^{t-1}.) For these specifications we have

$$F_t = 1 + r_{t+1}.$$

Using the CES utility functions, substituting in from the budget constraints, and rearranging equation (B.6) gives

$$a_2^{t-1} \geq \frac{\beta^\sigma F_t^\sigma (w_t + a_1^{t-1}(1+r_t) + B^{t-2} - \tau_t) + B^{t-1} - w_{t+1} + \tau_{t+1}}{(1+r_{t+1}) + \beta^\sigma F_t^\sigma}. \tag{B.7}$$

Thus we have three equations ((B.2), (B.3), and (B.7)) in three unknowns (B^{t-2}, a_1^t, and a_2^{t-1}) that must be satisfied each period.

REFERENCES

Altig, David and Steven J. Davis. 1989. "Government Debt, Redistributive Fiscal Policies, and the Interaction between Borrowing Constraints and Intergenerational Altruism." *Journal of Monetary Economics* 24 (1):3-30.

Auerbach, Alan J., and Laurence J. Kotlikoff. 1987. *Dynamic Fiscal Policy*, Cambridge University Press.

Auerbach, A.J., Laurence J. Kotlikoff, Robert Hagemann and Giuseppe Nicoletti. 1989. "The Economic Dynamics of an Aging Population: The Case of Four OECD Countries." *OECD Economic Studies* 12 (1):97-130.

Avery, R. B., and M. S. Rendall. 1994. *Estimating the Size and Distribution of the Baby Boomer's Prospective Inheritances.* Mimeo, Cornell University.

Barro, Robert J. 1989. "The Ricardian Approach to Budget Deficits." *Journal of Economic Perspectives* 3 (2):37-54.

Batina, Raymond G. 1987. "The Consumption Tax in the Presence of Altruistic Cash and Human Capital Bequests with Endogenous Fertility Decisions." *Journal of Public Economics* 34 (3):329-54.

Batina, Raymond G. 1999. "A Differential Incidence Analysis of a Tax Reform from an Income Tax to a Consumption Tax in the Presence of Bequests." *Public Finance Review* 27 (3):353-70.

Bergstrom, Theodore C., Lawrence Blume, and Hal Varian. 1986. "On the Private Provision of Public Goods." *Journal of Public Economics* 29 (1):25-49.

Bernheim, B. Douglas. 1991. "How Strong are Bequest Motives? Evidence Based on Estimates of the Demand for Life Insurance and Annuities." *Journal of Political Economy* 99 (5):899-927.

Bernheim, B. Douglas and Kyle Bagwell. 1988. "Is Everything Neutral?" *Journal of Political Economy* 96 (2):308-38.

Bernheim, B. Douglas, Andrei Shleifer and Lawrence H. Summers. 1985. “The Strategic Bequest Motive.” *Journal of Political Economy* 93 (6):1045-76.

Bruce, Neil and Michael Waldman. 1990. “The Rotten-Kid Theorem Meets the Samaritan’s Dilemma.” *Quarterly Journal of Economics* 105 (1):155-65.

Caballé, Jordi. 1995. “Endogenous Growth, Human Capital and Bequests in a Life Cycle Model.” *Oxford Economic Papers* 47 (1):156-81.

Conway, Karen S. 1999. “Are Workers Ricardian? Estimating the Labor Supply Effects of State Fiscal Policy.” *Public Finance Review* 27 (2):160-93.

Cremer, Helmuth and Pierre Pestieau. 1993. “Education for Attention: A Nash Bargaining Solution to the Bequest-as-Exchange Model.” *Public Finance* 48:S85-S97.

Cremer, Helmuth and Pierre Pestieau. 2003. *Wealth Transfer Taxation: A Survey.* The Levy Economics Institute Working Paper No. 394.

Evans, Paul. 1993. “Consumers Are Not Ricardian: Evidence from Nineteen Countries.” *Economic Inquiry* 31 (4):534-48

Futagami, Ritsuko, Kimiyoshi Kamada and Takashi Sato. 2004. “Government Transfers and the Samaritan’s Dilemma in the Family.” *Public Choice* 118 (1-2):77-86

Gale, William G. and Maria Perozek. 2001. “Do Estate Taxes Reduce Saving?” in *Rethinking Estate and Gift Taxation*. William G. Gale, James R. Hines and Joel B. Slemrod, Eds. Washington, D.C.: The Brookings Institution.

Gale, William G. and Joel B. Slemrod. 2000. “Life and Death Questions about the Estate and Gift Tax.” *National Tax Journal.* 54 (4):889-912.

Kaplow, Louis. 2001. "A Framework for Assessing Estate and Gift Taxation." In *Rethinking Estate and Gift Taxation*. William G. Gale, James R. Hines and Joel B. Slemrod, Eds. Washington, D.C.: The Brookings Institution.

Kocherlakota, Narayana R. 2001. "Looking for Evidence of Time-Inconsistent Preferences in Asset Market Data." *Quarterly Review*, Federal Reserve Bank of Minneapolis, 25 (3):13-24

Kotlikoff, Laurence J., Assaf Razin and Robert W. Rosenthal. 1990. "A Strategic Altruism Model in which Ricardian Equivalence Does Not Hold." *The Economic Journal* 100 (403):1261-8.

Kopczuk, Wojciech and Joel Slemrod. 2001. "The Impact of the Estate Tax on Wealth Accumulation and Avoidance Behavior." In *Rethinking Estate and Gift Taxation*. William G. Gale, James R. Hines and Joel B. Slemrod, Eds. Washington, D.C.: The Brookings Institution.

Kuehlwein, Michael. 1993. "Life-Cycle and Altruistic Theories of Saving with Lifetime Uncertainty." *Review of Economics and Statistics* 75 (1):38-47.

Kydland, Finn E. and Edward C. Prescott. 1988. "The Workweek of Capital and its Cyclical Implications." *Journal of Monetary Economics* 21 (2/3):343-60.

Laitner, John. 1993. "Long Run Equilibria with Borrowing Constraints and Altruism." *Journal of Economic Dynamics and Control* 17 (1-2):65-96.

Laitner, John. 2001. "Inequality and Wealth Accumulation: Eliminating the Federal Gift and Estate Tax." In *Rethinking Estate and Gift Taxation.* William G. Gale, James R. Hines and Joel B. Slemrod, Eds. Washington, D.C.: The Brookings Institution.

Laitner, John and F. Thomas Juster. 1996. "New Evidence on Altruism: A Study of TIAA-CREF Retirees." *American Economic Review* 86 (4):893-908.

Lambrecht, Stephane. 2003. "Altruistic Bequests and Non-negative Savings." *Louvain Economic Review* 69 (4):349-69.

Lucas, Robert. 1986. “Principles of Fiscal and Monetary Policy.” *Journal of Monetary Economics* 17 (1):117-34.

McGarry, Kathleen and Robert F. Schoeni. 1994. *Transfer Behavior: Measurement and the Redistribution of Resources within the Family.* National Bureau of Economic Research, Working Paper No. 4607.

Modigliani, Franco. 1988. “The Role of Intergenerational Transfers and Life Cycle Saving in the Accumulation of Wealth.” *Journal of Economic Perspectives* 2 (Spring):15-40.

Nishiyama, Shinichi. 2002. “Bequests, Inter Vivos Transfers, and Wealth Distribution.” *Review of Economic Dynamics* 5 (4):892-931.

O’Connell, Stephen A. and Stephen P. Zeldes. 1993. “Dynamic Efficiency in the Gifts Economy.” *Journal of Monetary Economics* 31 (3):363-79.

Rebelein, Robert P. 2002. “Ricardian Equivalence Survives Strategic Behavior.” *Public Finance/Finances Publiques* 53 (2):195-228.

Seater, John J. 1993. “Ricardian Equivalence.” *Journal of Economic Literature* 31 (1):142-90.

Smetters, Kent. 1999. “Ricardian Equivalence: Long-Run Leviathan.” *Journal of Public Economics* 73 (3):395-421.

2005 Economic Report of the President. 2005. U.S. Government Printing Office, Washington, D.C.

ENDNOTES

[1] Econlit offers only 13 entries for “Ricardian Equivalence" in 2003, one of the lowest numbers since 1986. Entries rose to 25 in 1992, fell for several years, then jumped to 26 in 1998. There are only five entries for “Ricardian Equivalence" in 2004 as of this writing.

[2] Conway (1999) finds that workers respond in a “Ricardian" manner to changes in state fiscal policy variables, particularly when states rely heavily on income taxes. However, a much more common result is illustrated by Evans (1993) (for example) which finds little evidence of Ricardian behavior in macro-level data. See Barro (1989) and Seater (1993) for surveys of the conditions believed necessary for Ricardian equivalence and their respective significance. Each also provides reviews of the micro and macroeconomic studies that test for empirical evidence of Ricardian equivalence.

[3] After running modest surpluses from 1998-2001, the total U.S. federal budget deficit soared to nearly 4 percent of GDP in 2004. Several European countries, including Germany and France, have exceeded the EMU-required deficit limit for the last several years, which necessitated relaxing the penalties originally prescribed by the Maastrict Treaty.

[4] Estimates of the portion of wealth accumulated for bequests vary widely, but the extremes range from 80 percent (Bernheim, Shleifer and Summers, 1985) to 20 to 30 percent (Modigliani, 1988).

[5] One benefit of examining causes of crowding out is to design policies that encourage private savings. For example, Batina (1999) illustrates the importance of incorporating potential saving-for-bequest motives to understanding the effects of shifting to a consumption tax.

[6] Bernheim, Shleifer, and Summers (1985) suggest a desire for child-to-parent services (e.g. phone calls, frequent visits, etc.), rather than altruism, motivates parental transfers. The true motivation for parent-to-child transfers remains an open question in economics and is not an issue addressed in this paper. Bernheim (1991) offers additional discussion on this question. Altruism receives substantial attention in the literature hence it is the motivation used in this paper.

[7] Others have also used versions of the Samaritan’s dilemma to examine the effects of government policies and intrafamily linkages. For example, Futagami, Kamada, and Sato (2004) employs a similar version of the Samaritan’s dilemma in an examination of government transfer policies. Lambrecht (2003) analyzes some of the dynamics of using the Samaritan’s dilemma to model intergenerational relationships. Gale and Perozek (2001) incorporate the Samaritan’s dilemma into a two-period, parent-child model to examine a potential role for public parent-to-child transfers as a means of eliminating the child’s incentive to act manipulatively.

[8] For example, Barro (1989) closes with the prediction that "the Ricardian approach will become the benchmark model for assessing fiscal policy."(p.52)

[9] Lucas (1986) suggests distortionary taxes may be the primary source of deviations from Ricardian equivalence. (p.121)

[10] An illuminating empirical analysis by Kopczuk and Slemrod (2001) finds a slight negative relationship between estate tax rates and the size of reported estates. The theoretical work in this area continues to be divided. For example, Laitner (2001), in a detailed general-equilibrium analysis, finds that *removing* estate taxes will increase aggregate saving. In contrast, using partial-equilibrium analysis, Gale and Perozek (2001) conclude *imposing* estate taxes can actually increase aggregate saving, a result that is examined further below.

[11] Table 4 in Cremer and Pestieau (2003) summarizes the changes expected in response to a variety of fiscal policies for altruism and other bequest motives.

[12] In a more general version of this model I allow inter vivos transfers in addition to end-of-life bequests. Because strategic behavior leads consumers to squander resources when young, parents always do best by not giving an inter vivos transfer. Thus prohibiting inter vivos transfers has no effect on the results of this analysis. While this may conflict with observations of middle-aged parents providing support to young children such transfers are generally tied to particular uses (such as education) rather than available for general consumption. Also, because one period in this model spans 20 years, inter vivos transfers made late in life are effectively combined with end-of-life bequests.

[13] The intergenerational discount rate indicates the weight put on the child's utility in the parent's utility function. For example, a value of 0.5 indicates the parent values his child's utility only 50 percent as much as the parent values his own utility from consumption. Then, in equilibrium, the parent seeks to equate his child's marginal utility of consumption to twice his own marginal utility of consumption.

[14] Some authors have chosen a modification of this approach. For example, Caballé (1995), Batina (1987), and Cremer and Pestieau (1993) each use a three-period overlapping generations model and assume a parent makes all of his child's decisions for the child's first period of life, thus effectively removing the child's opportunity for manipulation.

[15] Parents may appear to commit to a bequest amount by writing a will. In fact a parent is also free to subsequently change the will, and thus is not really committing to a bequest amount.

[16] Kockerlakota (2001) finds no evidence that consumers have time-inconsistent preferences in his examination of asset prices.

[17] A third possibility exists in that the child could make her choices before the parent makes his choices. I omit this case because it seems highly unlikely a parent considering a bequest would wait to make his savings decisions until the child made all of her pre-bequest decisions.

[18] By assumption we are interested only the case of positive bequests. Then, since individuals can borrow or save and $\lim_{c\to 0} u'(c) = \infty$, all budget constraints and first order conditions are satisfied with equality.

[19] Appendix B gives a specific derivation of this function for the computer simulations.

[20] In the simultaneous-choice specification the young consumer chooses savings a_1^t at the same time as the middle-aged consumer chooses savings a_2^{t-1}. Later, in the sequential-choice specification, the middle-aged consumer chooses savings *before* the young consumer acts so these functions will not be dependent on a_1^t in that specification.

[21] See Appendix B for details on the derivation of this result.

[22] An analogous result is presented by Bergstrom, Blume, and Varian (1986). They demonstrate that a wealth redistribution amongst voluntary contributors to a public good has no effect of the provision of the public good. In my model the child's utility is a pure public good – both parent and child enjoy the child's utility non-rivalrously and without possibility of exclusion. The change in the timing of lump sum taxes is a simple redistribution across generations.

[23] Altig and Davis (1989) uses a similar measure.

[24] I also calculate *R* values for the case of immediate debt repayment and for the case when debt is left outstanding indefinitely. Not surprisingly, the values are significantly smaller when debt repayment is immediate.

[25] The "max" operators on the right hand side of the following expressions reflect the fact that consumers may borrow as well as save, but capital gains taxes are collected only on returns to savings.

[26] The specific parameter ranges are as follows: *A* varies from 1 to 5; β varies from 0.25 to 0.65 (corresponding to annual rates of approximately 0.93 to 0.98); γ varies from -0.5 to -4; ρ varies from 0.05 to 0.15; and the portion

of government expenditures financed with debt varies from 10 percent to 30 percent.

[27] These results differ from those obtained by Bernheim and Bagwell (1988) because they assume the tax an individual owes in a period to be independent of the person's actions in the period, thereby eliminating the distortionary nature of the labor, capital, and inheritance taxes. Note, however, that a person's behavior may influence the taxes he pays in the future.

[28] Two other effects, the small tax increase required to pay interest on the debt and the future tax increase required to retire the debt, could also be included in this list. However, the former has only modest effects and the latter only affects consumers a few periods before it occurs.

[29] As Kaplow (2001, 188) points out, "many transfers, including bequests, come out of savings derived from previous receipts." Kaplow continues by pointing out that bequests are merely a type of end-of-life consumption. Then, "A tax on a subset of [final-period] consumption can be viewed in part as a tax on [final-period] consumption and hence as a tax on savings." (Kaplow 2001, 188). Thus we expect qualitative similarities between the effects of inheritance and capital gains taxation.

[30] When altruism is the motive for intergenerational transfers, a parent gains utility when his child's utility increases, other things held constant. The giving of a bequest is therefore equivalent to purchasing a good. When the parent's income increases he will both purchase more of his own consumption goods and give a larger bequest, effectively purchasing additional units of his child's utility. If the parent receives the additional income prior to the period in which the bequest is given he will save more in order to finance the larger bequest.

[31] Gale and Perozek (2001) use a partial equilibrium model with intergenerational altruism and strategic behavior similar to the type used in this paper. They find that lowering estate tax rates causes bequests to increase but the larger bequest amounts may cause the child's saving to decrease more than the parent's saving increases, thereby causing a net decline in aggregate saving. Unfortunately, their partial equilibrium framework fails to account for the persistence of larger bequests observed in this general equilibrium analysis. In their model, the parent benefits from the tax decrease, but does not also receive the additional income from a larger bequest. The child receives a larger bequest but, since the child has no child of her own to give a bequest to, she has only her own future consumption to motivate higher saving. Absent these two reasons for savings increases, their analysis likely understates the true change in aggregate saving that occurs as a result of a temporary, debt-financed, tax decrease.

[32] 2005 Economic Report of the President, Table B-34, p. 251.

[33] 2005 Economic Report of the President, Table B-78, p. 303 and Table B-84, p. 309. Annual numbers were adjusted using GDP deflator values from Table B-3, p. 212, of the same source.

[34] ibid.

[35] The units here are billions of dollars per thousands of people per 20 years. This is equivalent to an annual per capita figure of $4,051.

[36] This corresponds to an annual per capita tax cut of $878.

[37] See, for example, Laitner and Juster (1996) and McGarry and Schoeni (1994).

[38] Two things to be aware of with the inheritance data from the PSID: first, the PSID doesn't discriminate between bequests given by parents from those given by others. Respondents were asked in 1988 (if and) when their parents died, but this question was discontinued the following year. Second, the IRS reported hundreds of bequests by parents to children in excess of $1 million in 1989. The PSID apparently underreports transfers by higher-income decedents. Since these two problems affect the bequest estimate in opposing directions there is reason to expect they are somewhat offsetting and that the estimate above is reasonably accurate.

[39] The reader may notice that the intergenerational discount rate (ρ) used in the example in the text is higher than that determined here via calibration. This occurs because, for the example, government spending and taxes are set to zero. In the calibration, first-period savings falls because of the taxes the young consumer must pay. To compensate elderly consumers increase bequests to their middle-aged children. Thus second period savings rises and bequests by subsequent generations rise further. (These are the changes predicted by Ricardian equivalence.) Other parameters held constant, the steady-state bequest amount is considerably larger when taxes are included than when they are absent. In order to produce positive bequest amounts in the example (particularly in the non-manipulative specification), the intergenerational discount rate must be greater than the rates determined here.

[40] Total U.S. GDP from 1985-2004 was $164,036.6 billions of chained 2000 dollars. Source: 2005 Economic Report of the President, Table B-2, p. 210.