

SPECIFICATION AND ESTIMATION OF A
GENERALIZED CORNER SOLUTION MODEL
OF DEMAND: AN ANEMIYA-TOBIN APPROACH

Edward R. Morey*, Donald Waldman*,
Djeto Assane**, and W. Douglass Shaw***

Vassar College Economics Working Paper # 4

December, 1989

- * Department of Economics, University of Colorado
- ** Department of Economics, University of Nevada
- *** Department of Economics, Vassar College

Vassar College Economics Working Paper # 4

Introduction

In this paper we specify a stochastic behavioral model of consumer allocation that admits the possibility that a subset of commodities will not be consumed. Our motivation is the widespread and increasing availability of large microdata sets with information on detailed expenditure categories. In such data sets, it is frequently the case that one or more commodities are not consumed. The proposed model leads to a system of desired and observed share equations that are both consistent with economic theory and estimable.

The idea of estimating a complete system of demand or share equations has fascinated economists for decades, and in fact has led to the introduction of many well-known innovative econometric techniques. Since, until recently, the typical application involved aggregate data, shares to the several aggregates could always be found on the unit interval and attention was focused on such estimation problems (unique to economics) such as the constraining and testing of regression parameters according to various adding up restrictions.

However, in recent years several developments have broadened the range of econometric issues to be dealt with. One is the aforementioned availability of micro data sets with detailed expenditure categories. In addition, economists have become interested in applying the consumer demand framework to nontraditional situations such as time allocation, and budget allocation over discrete commodity bundles.¹ As a result of these developments, observed budget shares are now frequently zero, and even equal to one in some applications. This situation presents no new theoretical problems from a deterministic perspective, as the Kuhn-Tucker conditions easily allow for these general corner solutions. But a new problem in estimation arises as the usual

¹See, for econometric examples, Dubin and McFadden 1984, Hausman and Wise 1984.

stochastic assumptions regarding regression errors are no longer appropriate, much the same as in many discrete choice models.

Considering this stochastic problem at a fundamental level, Tobin (1958) formulated a model where a single good may or may not be purchased, and if purchased may be purchased in variable amounts. This was extended to a system of goods by Amemiya (1974), and to a system of observed shares constrained between zero and one that satisfies a budget constraint by Wales and Woodland (1983, hereafter "WW"). However, these methods are all based upon normally distributed errors added to share or demand equations. In particular, WW assume that desired shares are normally distributed. To make the observed shares compatible with the stochastic model, shares outside the unit simplex are truncated to zero.² The assumption that desired shares are normally distributed is problematic. A normally distributed random variable is unbounded, while shares are, of course, bounded to the unit interval.^{3,4}

In this paper we attempt to improve upon the Amemiya-Tobin approach of WW by assuming a probability distribution for the desired and observed shares

²In the same paper, WW also attack the problem of zero shares with a second approach, by assuming a utility function with random parameter variation that leads to an estimable form of the Kuhn-Tucker conditions. Lee and Pitt (1986, 1987) also adopt a Kuhn-Tucker approach. They develop what amounts to the dual approach to the Kuhn-Tucker method of WW and point out the advantages of the use of indirect utility functions in this context, for example, the ability to decompose the effect of price changes into a direct effect and a "regime switching" effect.

³Note that normally distributed demand is equally problematic. Demand is also bounded, it cannot be negative and total expenditures must exhaust the budget.

⁴Results from Woodland (1979) provide some justification for continuing to assume that shares are normally distributed if all the observed shares in the data set are strictly in the interior of the unit simplex. However, Woodland (1979) does not consider situations where many zero shares are observed, and his results cannot be used to justify normality assumptions in these situations.

that, unlike the normal, is consistent not only with observed shares but also with economic theory. We propose the use of the Dirichlet distribution, first suggested in an earlier paper by Woodland (1979), for the probability distribution of desired shares. However, the individual cannot always consume their desired consumption bundle. The individual might desire to purchase a commodity in a quantity that is smaller than the minimum quantity that is available in the market place. In such cases, we assume the share is truncated to zero and the budget reallocated. With these assumptions, both desired and observed shares lie on the unit simplex.

In the next section, this two-stage allocation model is presented. At the first stage, it is assumed that consumers allocate their budget to the various commodities without regard to the minimum consumption constraints imposed by the marketplace. In the second stage, any desired shares that are "too small" are truncated to zero and the released funds are reallocated. This two-stage procedure can be consistent with overall utility maximization if the process of optimization is costly. Econometric implementation is discussed in section III, and the model is then applied to a data set containing information on the budget shares allocated to various recreational sites.

I. A Generalized-Corner Solution Model of Consumer Behavior

Consumers consider a vector of J commodities, $x = [x_j]$, $j = 1, 2, \dots, J$; where x_j is the quantity of commodity j demanded by individual i .⁵ Individual preferences are represented by the direct random utility function

⁵ The individual specific subscript is omitted in this section for reasons of notional simplicity. Commodities can be viewed as either market goods and services, or as activities that are produced and consumed by the individual.

$$(1) U = U(x, c, a, \epsilon),$$

where

$c \equiv [c_m]$, $m = 1, 2, \dots, M$; is a vector of measurable characteristics of the individual.

$a \equiv [a_{kj}]$, $k = 1, 2, \dots, K$; is a vector of characteristics the individual associates with commodity j , and where these K magnitudes are observed by both the individual and the investigator; and

$\epsilon \equiv [\epsilon_j]$, represents those characteristics known to the individual but not observed by the investigator. Therefore, each ϵ_j is deterministic from the individual's perspective but a random variable from our perspective.

A two-stage optimization procedure is assumed. At the first stage, the individual determines his desired consumption bundle by maximizing $U(x, c, a, \epsilon)$ subject to the constraint that $Y \geq p'x$, where Y is the budget allocation to the J commodities and $p = [p_j]$, where p_j is the price of commodity j .⁶

The desired bundle is assumed to contain a strictly positive amount of each commodity. This assumption is only weakly restrictive; it simply means that the individual would like to consume at least an infinitesimal amount of each commodity. We, for example, may desire to purchase, at prevailing prices, an hour of a Pavarotti opera, a frame of bowling, and a sip of 1961 Chateau Lafite-Rothschild, but for every commodity there is a limit to how little can be purchased. This will sometimes cause desired and actual consumption to diverge.

⁶ If the commodity is an activity, p_j is the cost of producing one unit of the activity.

The desired consumption bundle can be represented by either a system of desired demand equations or by a system of desired share equations. Let x_j^* represent the desired demand for commodity j where

$$(2) \quad x_j^* = x_j^*(p, Y, c, a, \epsilon) \quad j = 1, 2, \dots, J.$$

Let s_j^* represent the desired share for commodity j where⁷

$$(3) \quad s_j^* = s_j^*(p, Y, c, a, \epsilon) \quad j = 1, 2, \dots, J.$$

Both x_j^* and s_j^* are deterministic from the individual's perspective but random variables from our perspective. Denote the expectation of x_j^* as \hat{x}_j , and the expectation of s_j^* as \hat{s}_j . The expected desired bundle $(\hat{x}_j, j=1, 2, \dots, J)$ is the solution to $\{\max u(x, c, a) \text{ s.t. } Y \geq p'x; \text{ where } u(x, c, a) = U(x, c, a, \underline{Q})\}$.

For our purposes, the desired bundle, and its expectation, are most conveniently represented in share form because the share representation makes the stochastic properties of the desired bundle transparent. The desired shares must have the following properties: $0 \leq s_j^* \leq 1 \forall j$ and $\sum s_j^* = 1$.

⁷ Note that these desired shares can represent either proportion shares or expenditure shares; i.e.,

$$s_j^* = x_j^* / [\sum_{\ell} x_{\ell}^*] \quad j = 1, 2, \dots, J. \quad \text{if they are proportion shares}$$

or

$$s_j^* = p_j x_j^* / [\sum_{\ell} p_{\ell} x_{\ell}^*] \quad j = 1, 2, \dots, J. \quad \text{if they are expenditure shares}$$

The choice is arbitrary for our analysis. When commodities are defined as goods it is common to work with expenditure shares. When commodities are defined as activities it is common to work with either proportion or expenditure shares.

In practice, the individual cannot always consume their desired consumption bundle. The individual might desire to consume a commodity in a quantity that is smaller than the minimum quantity available in the market place. At the second-stage, the individual is assumed to take account of these minimum consumption constraints by truncating to zero those desired shares that fall below some specified amount and proportionally expanding the remaining shares until they sum to one. The result is that the individual's observed shares are

$$\begin{aligned}
 s_j &= 0 && \text{if } s_j^* \leq s_m(j); && \text{and} \\
 s_j &= s_j^* / \left(\sum_{\ell \in S} s_\ell^* \right) && \text{if } s_j^* > s_m(j)
 \end{aligned}
 \tag{4}$$

where $S = \{j: s_j^* > s_m(j)\}$.

These $s_m(j)$ functions can be made as simple or as complicated as appropriate. For example, one might assume that $s_m(j) = s_m \forall j$. Such an assumption would be appropriate if the individual decides it is too costly to determine the minimum amount that can be consumed of each commodity so simply adopts a rule of thumb that says any commodity whose desired share is less than some amount will not be consumed. Alternatively, it might be assumed that each commodity has its own s_m , $s_m(j)$, where $s_m(j)$ is the share that would allow the individual to purchase one standard unit of the commodity j . For example, one cannot purchase less than a full game of bowling and tickets are not sold for fractions of operas.

Summarizing, this two-step choice algorithm suggests a decision process of the following sort. The individual first decides what he would like to

consume ignoring the fact that for many goods it is impossible to consume them in less than some minimum amount. The individual has learned that by ignoring these minimum consumption constraints he can greatly reduce the complexity of his choice problem and still obtain a good first approximation to the solution to the more complex problem. Then, depending on the complexity of his s_m rule (which has also been determined by experience), he either chooses to not consume all the commodities whose desired shares are below some common critical magnitude, or he decides, on a commodity by commodity basis, to consume none of a commodity if his desired share for that commodity is not sufficient to purchase a standard unit of that commodity.⁸

Visualizing the decision process, the individual's choice set at the first-stage of the decision process can be represented, in share form, as a right-angled $(J - 1)$ dimensional polyhedron that is of unit length in each of the $(J - 1)$ dimensions; i.e., it is the $(J - 1)$ unit simplex. The shares for the first $J - 1$ commodities are the coordinates on the $J - 1$ orthogonal axes while the share of the J th commodity is the residual. If, for example, there are four commodities, the individual's vector of desired shares can be represented as a point in the interior of the outer tetrahedron in Figure 1. The inner tetrahedron in Figure 1 lies s_m units inside of the outer tetrahedron in each of the $(J - 1)$ dimensions. The individual will truncate to zero at least some of his $(J - 1)$ desired shares if his vector of $(J - 1)$ desired shares lies

⁸ Alternatively, one might assume that the individual accounts for the minimum consumption constraints from the very beginning. Such an assumption implies that the individual determines his consumptions bundle in one step as the solution to a deterministic integer programming problem which will be stochastic from our perspective. This alternative assumption, while possibly appealing to the optimization purist, is not necessarily more appropriate and is definitely more difficult to model and estimate than the two-stage decision process outlined in the text.

in a part of the outer tetrahedron that is not contained in the inner tetrahedron. For example, if point A represents the individual's desired shares, the individual will not truncate and observed shares equal desired shares. However, if the individual's desired shares are represented by any point on line segment C'C, $s_4 < s_m$ (that is, the desired share of the fourth commodity is less than the minimum consumable amount), the individual does not consume the fourth commodity and expands his budget shares of the first three commodities in proportion to their desired levels. This amounts to projecting the ray OC' to the surface of the outer tetrahedron, at point C. There, the sum of the first three observed shares is one.

II. Stochastic Assumptions and the Likelihood

To derive maximum likelihood (ML) estimates of the parameters of the utility function (equation 1) from information on shares of commodities consumed (and not consumed) by individuals, stochastic assumptions must be made. The traditional approach is to assume that the observed shares are multivariate normally distributed with constant covariance matrix and means depending upon prices and individual characteristics. But the normal distribution is an untenable assumption for these random variables because the observed shares are, by definition, bounded between zero and one, and must sum to one. For micro data sets, there will also be numerous observed shares of zero. Persuasive arguments to this effect are made by Woodland (1979, pps. 361-363), who then assumes the observed shares follow the Dirichlet distribution. While the Dirichlet specification for the observed shares is an improvement over the normal, since values are constrained between zero and one, it is not consistent with observed shares of zero. This can be a significant liability because in

most micro applications few people consume positive quantities of all commodities.

Rather than assuming the observed shares follow the Dirichlet distribution, in this paper it is assumed that the $J - 1$ desired shares follow the Dirichlet distribution:

$$(5) \quad f(s_1^*, \dots, s_{J-1}^*) = k (1 - \sum_{j=1}^{J-1} s_j^*)^{\hat{x}_J - 1} \prod_{j=1}^{J-1} s_j^{* \hat{x}_j - 1}$$

where $k = \Gamma(\sum_{j=1}^{J-1} \hat{x}_j) / \prod_{j=1}^{J-1} \Gamma(\hat{x}_j)$ and $\Gamma(\cdot)$ is the gamma function.

This assumption, in conjunction with the behavioral rule that maps desired shares into observed shares, admits observed shares of zero.

This approach is in the tradition of WW who assume that the desired shares are multivariate normally distributed and then map desired shares outside the unit simplex onto the boundary of that simplex. However, desired shares cannot be multivariate normally distributed for the same reasons that observed shares cannot be multivariate normally distributed.

As a result of the s_m behavioral rule that maps desired shares into observed shares, the likelihood function necessarily involves the evaluation of some line and surface integrals of the Dirichlet function, in addition to evaluation of the more usual probability density functions. To understand the process of forming the likelihood, the case of $J=3$ shares is examined. Restricting the discussion to this case makes it possible to employ a graph in both desired and observed share space, similar to the graph of Section I. The extension to higher dimensions is conceptually, at least, straightforward.

Consider Figure 2. The shares of commodities 1 and 2 are graphed along the horizontal and vertical axes, respectively. The line connecting (1,0) and (0,1), has equation $s_1^* + s_2^* = 1$. The inner triangle lies s_m units inside of

the outer triangle. The horizontal line segment of the inner triangle has the equation $s_2^* = s_m$, the equation for its vertical line segment is $s_1^* = s_m$, and the equation for its hypotenuse is $s_1^* + s_2^* = 1 - s_m$. The desired shares for commodities one and two can be represented by a point inside the outer triangle. The desired share for the third commodity is computed as the residual, $s_3^* = 1 - s_1^* - s_2^*$.

Suppose the individual's desired shares for goods one and two are both greater than s_m , and they place him below and to the left of the line $s_1^* + s_2^* = 1 - s_m$. Point A represents just such a person. Then his desired share for the third commodity is also greater than s_m , and all three observed shares will be positive and equal to the desired shares. The contribution to the likelihood from this interior solution is then simply the value of the bivariate Dirichlet density function evaluated at the point (s_1, s_2) :

$$(6) \quad f(s_1, s_2) = k s_1^{(\hat{x}_1-1)} s_2^{(\hat{x}_2-1)} (1-s_1-s_2)^{(\hat{x}_3-1)}$$

This is simply the height of the density function above the point (s_1, s_2) .

Suppose, to take the other extreme case, the desired shares of commodities one and two are both less than s_m . In Figure 2, the point representing this person would be somewhere in the rectangular region bounded by the axes and the equations $s_1^* = s_m$, $s_2^* = s_m$, at a point such as C. Therefore, according to our behavioral rule, his entire budget would be spent on the third commodity. The probability of observing $s_1 = s_2 = 0$ and $s_3 = 1$ is then the joint probability that $s_1^* < s_m$ and $s_2^* < s_m$, given by

$$(7) \quad P(0 \leq s_1^* \leq s_m; 0 \leq s_2^* \leq s_m) = k \int_0^{s_m} \int_0^{s_m} s_1^{(\hat{x}_1-1)} s_2^{(\hat{x}_2-1)} (1-s_1-s_2)^{(\hat{x}_3-1)} ds_1 ds_2$$

In terms of Figure 2, this is the area under the Dirichlet density above the square formed by the axes and the lines $s_1^* = s_m$ and $s_2^* = s_m$. The corner solutions for $s_1 = 1$ and $s_2 = 1$ follow analogously.

Now suppose that the individual demands enough of both the first and second commodity (with the demand for each greater than s_m) so that the third commodity's share falls below s_m . His desired demands would place him at a point such as D^* in the figure, since at D^* the sum of s_1^* and s_2^* is greater than $1 - s_m$. But what would observed shares be for these desired shares? According to the proportional expansion rule of section II, the individual does not consume commodity three and divides his desired expenditures on commodity three between commodities one and two in the same ratio as his desired shares for commodities one and two. Therefore his observed shares would lie on a ray from the origin through point D^* at the intersection of the line $s_1^* + s_2^* = 1$, at point D'' , where all his income is exhausted. But notice that, by the same line of reasoning, any point representing desired shares on the line segment $D'D''$ would be similarly mapped onto the point D .

The increment to the likelihood function for any individual whose observed shares lie at point D'' is the line integral along OR under $f(s_1^*, s_2^*)$ and between D' and D'' . This line integral takes the general form:

$$(8) \quad f(D^*) = \int_R f[x(t), y(t)] \sqrt{\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2} dt$$

The coordinates $x(t)$ and $y(t)$ are parametrically defined as

$$s_1 = x(t) = t$$

$$s_2 = y(t) = \frac{s_2}{s_1} t \quad \text{i.e., } s_3 = 1 - t - \frac{s_2}{s_1} t$$

The quantity $\sqrt{\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2}$ is the length of $D'D''$ which for our problem simplifies to:

$$(9) \quad \sqrt{\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2} = \sqrt{1 + \left[\frac{s_2}{s_1}\right]^2} = \sqrt{(s_1^2 + s_2^2)/s_1^2} \stackrel{\text{def.}}{=} s$$

given that $\frac{dx}{dt} = 1$ and $\frac{dy}{dt} = \frac{s_2}{s_1}$.

To find the limit of integration defined over the region R , we observe from Figure 2 that the boundary line can be represented by $x+y = 1 - s_m$ with $y = \frac{s_2}{s_1}x$, $\frac{s_2}{s_1}$ being the slope of line $D'D''$ along OR . Solving simultaneously, the two equations give the limit of integration expressed in share form as: $[s_1(1-s_m), s_1]$. $f(D^*)$ can now be written as:

$$(10) \quad f(D^*) = \int_{s_1(1-s_m)}^{s_1} f\left(t, \frac{s_2}{s_1}t\right) s dt$$

Given that shares are Dirichlet distributed, (10) is explicitly written as:

$$(11) \quad f(D^*) = \int_{s_1(1-s_m)}^{s_1} k t^{(\hat{x}_1-1)} \left(\frac{s_2}{s_1}t\right)^{(\hat{x}_2-1)} \left[1-t-\frac{s_2}{s_1}t\right]^{(\hat{x}_3-1)} S dt$$

Factoring out some constant terms, we have

$$(12) \quad f(D^*) = k S \left(\frac{s_2}{s_1}\right)^{(\hat{x}_2-1)} \int_{s_1(1-s_m)}^{s_1} t^{(\hat{x}_1+\hat{x}_2-2)} \left[1-t-\frac{s_2}{s_1}t\right]^{(\hat{x}_3-1)} dt$$

Simplifying the integrand:

$$(13) \quad t^{(\hat{x}_1+\hat{x}_2-2)} \left[1-t-\frac{s_2}{s_1}t\right]^{(\hat{x}_3-1)} = t^{(\hat{x}_1+\hat{x}_2-2)} \left[1-\left(\frac{s_1+s_2}{s_1}\right)t\right]^{(\hat{x}_3-1)}$$

$$= t^{(\hat{x}_1 + \hat{x}_2 - 2)} [1 - ct]^{(\hat{x}_3 - 1)}$$

where $c = \frac{s_1 + s_2}{s_1} = \frac{1}{s_1}$ since from D' to D'' , $s_1 + s_2 = 1$.

Thus, (12) simplifies to:

$$(14) \quad f(D^*) = kS \left(\frac{s_2}{s_1} \right)^{(\hat{x}_2 - 1)} \int_{(1-s_m)s_1}^{s_1} t^{(\hat{x}_1 + \hat{x}_2 - 2)} [1 - ct]^{(\hat{x}_3 - 1)} dt$$

The integral in equation 14 is a function of the variable t and is very close to a beta function. Further transformations necessary to put equation 14 in a form suitable for computation are discussed in the Appendix.

The likelihood for a sample of n individuals is the product of the probability expressions for the three types of individuals, equations (6), (7) and (14), and it is maximized with respect to choice of \hat{x}_i .

III. An Application: Demand for Site-Specific Recreational Activities

For purposes of illustration, the model is used to explain how individuals allocate their time among site-specific recreational activities. Recreational demand data indicates that different individuals consume different subsets of the available sites (commodities). Interior points (individuals who visit all of the sites) are rare; "corners" (individuals who visit a single site) and "boundaries" (individuals who visit two or more but not all of the sites) predominate. However, no one, to our knowledge, has estimated a model

that takes this aspect of the data into account.⁹ All the estimated recreational demand models either assume interior solutions or constrain the individual to a corner.

The observation that recreators do not, in general, consume all of the commodities is not unique to recreational data; i.e., most individuals do not consume some of every alternative. Often, this is even true when the alternatives are broadly defined.¹⁰ Therefore, the model should be well suited to estimating individual demand functions even when the number of alternatives is small.

The model is used to explain how anglers in upstate New York allocate their time among three popular fishing sites (Lake George, Great Sacandaga Lake and Lake Saratoga). A data set was constructed by Morey and Shaw (1989) from data collected by the State of New York. The data set contains 459 individuals who visited at least one of these sites. Only 1.5% of this sample visited all three sites; 7.5% percent visited only sites 1 and 2; 3% visited only sites 1 and 3; 4% visited only sites 2 and 3; 30% only visited site 1; 33% only visited site 2; and 21% only went to site 3. There is information for each individual on the number of days spent at each site, fish catch, income, ability level (novice, intermediate and advanced), species preference and the location of residence.

The cost of a trip to site j by individual i , p_j^i , includes travel costs, equipment costs and the opportunity cost of the individual's time both in

⁹ Bocksteal, Hanemann and Strand (1984) in their excellent survey on recreational demand modelling discuss, at length, the fact that most recreators only visit a subset of the available sites. They suggest a Kuhn-Tucker approach in the tradition of WW but they do not estimate the model.

¹⁰ Everyone consumes food, clothing and shelter, but once the data are disaggregated, most individuals will not consume some of each subaggregate.

travel and on-site. These were calculated by Morey and Shaw (1989). Cost varies across sites, for a given individual, as a function of the site's location. They vary across individuals, for a given site, as a function of where the individual lives and the opportunity cost of their time. Average catch rates by species and ability level were calculated. The average catch data indicates that species availability varies substantially across the three sites, and that variations in average catch rates by ability level are substantial and vary across sites.

Assume that the deterministic component of preferences for fishing at the three sites can be represented by the CES utility function

$$(15) \quad u^i = u(x^i, a^i) = \sum_{\ell=1}^3 h(a_{\ell}^i) (x_{\ell}^i)^{\beta} \quad i = 1, 2, \dots, 459.$$

where $1 > \beta \neq 0$,

$$h(a_{\ell}^i) = [\alpha_0 + \alpha_1 a_{\ell}^i]^2 \quad \text{and}$$

a_{ℓ}^i = the average catch rate at site ℓ for those of individual i 's ability level and for the species indicated as most preferred by individual i .

The CES form was chosen because it well known, tractable, and easily allows for the incorporation of characteristics. For other applications of the CES to recreational demand modelling see Morey (1981 and 1984).

Given this CES form, the proportion of fishing days we expect individual i desires to spend at site j is

$$(16) \quad \hat{s}_j^i = \hat{s}(p_j^i, p^i, a_j^i, a^i) = \frac{1}{\sum_{\ell \in L} \left[\frac{[h(a_j^i) p_{\ell}^i]^{\sigma}}{p_j^i h(a_{\ell}^i)} \right]} \quad j = 1, 2, 3.$$

where $L = \{1,2,3\}$ and $-\sigma = -1/(1 - \beta)$ is the constant Hicks-Allen elasticity of substitution.¹¹ Since these share equations are homogeneous of degree zero in the α parameters, α_0 was set to 1 without loss of generality. This leaves two parameters to estimate, α_1 and σ . To complete the model, assume that $s_m = .1$; i.e., assume experience has taught the individual it is, in general, not feasible to visit a site if he desires to allocate less than 10% of his trips to that site.

The likelihood function for these expected desired shares was maximized for the sample.¹² The estimated parameter values (and asymptotic t statistics) are $\hat{\alpha}_1 = .112$ (203.5) and $\hat{\sigma} = .461$ (-286.7). Likelihood ratio statistics indicate that: (1), the model with prices only ($\alpha_1 = 0$) explains the allocations across sites significantly better than a model that assumes each individual randomly allocate their time across the sites; and (2), the model with both prices and catch rates explains the allocation across sites significantly better than the model with only prices.

The influence of these parameter estimates can be considered on three levels: (1) each individual's expected desired shares; (2) the probability that an individual will visit only a certain subset of sites; and (3) expected

¹¹Correspondingly, the number of fishing days we expect individual i desires to spend at site j is

$$\hat{x}_j = \hat{x}(p_j^i, p^i, a_j^i, a^i) = \frac{Y}{\sum_{\ell=1}^J p_\ell^i \left[\begin{array}{c} p_\ell^i h(a_j^i) \\ \vdots \\ p_j^i h(a_\ell^i) \end{array} \right]^{-\sigma}} \quad j = 1, 2, 3.$$

¹² The optimization package was GQOPT and a David-Fletcher-Powell algorithm was utilized.

shares conditional on the individual choosing a particular group of the sites. All of these estimated measures will vary across sites and individuals as a function of catch rates, species preference, residential location, site location and the opportunity cost of the individual's time.

Examining first the expected desired shares, the estimated Allen elasticity of substitution, $-.461$, indicates that a one percent increase in the price ratio for any two of the sites will lead to a $.46$ percent decrease in the ratio of the individual's expected desired demands for those two sites. However, this inelastic response in terms of expected desired demands could result in a significant change in observed demands if the change causes a desired share to move above or below s_m .

A one percent increase in the price of site j will lead to a $[\ .461*(1 - \hat{s}_j^i)]$ percent decrease in individual i 's expected desired share for site j , and a $.461*\hat{s}_j^i$ increase in the individual's expected desired share for site m , $m \neq j$. Examining the elasticities with respect to average catch rates, a one percent increase in the average catch rate at site j for the individual's most preferred species will lead to a $[\ .103*(1 - \hat{s}_j^i)/(1 + .112*a_j^i)]$ percent increase in the individual's expected desired share for site j . All of these price and catch elasticities will vary across individuals, for a given site, as a function of the individual's ability level, species preference, value of time and residential location. For a given individual, they will vary across sites as function of the site's location and species availability.

Turning now to observed, rather than desired, behavior, the model will not predict which group of sites each individual will choose to visit, but can predict, for each individual, the probability that the individual will choose a

particular group of the sites. For example, the estimated probability that individual i will visit all three sites is

$$(17) \quad \Pr(s_1 > s_m, s_2 > s_m, s_1 + s_2 < 1 - s_m) \\ = \int_{s_m}^{1-2s_m} \int_{s_m}^{1-s_m-s_1} f(s_1, s_2) ds_2 ds_1,$$

and the estimated probability that individual i will visit sites 1 and 2 only is

$$(18) \quad \Pr(s_1 > s_m, s_2 > s_m, s_1 + s_2 > 1 - s_m) \\ = \int_{s_m}^{1-s_m} \int_{s_m}^{1-s_1} \hat{f}(s_1, s_2) ds_2 ds_1 - \int_{s_m}^{1-2s_m} \int_{s_m}^{1-s_m-s_1} \hat{f}(s_1, s_2) ds_2 ds_1$$

where $\hat{f}(s_1, s_2)$ is given by equation (6) with the ML estimates of \hat{x}_i . As with the expected desired shares, the estimated probability that an individual will choose to consume a particular group will vary across individuals as a function of their ability level, species preference, value of time and location of residence.

It is also of interest to consider the expected shares conditional upon the choice of a particular group of sites. As noted above, one cannot predict which group of sites the individual will visit, but one can estimate the expected shares conditional on each possible outcome. For example, one can estimate the expected shares for sites 1 and 2 assuming that the individual visits only these two sites.

Notationally, define $c\hat{s}_j^i[1,2]$, $j = 1,2,3$ as individual i 's expected share for site j conditional on him choosing to visit only sites 1 and 2. Obviously, $c\hat{s}_3^i[1,2] = 0$ and $c\hat{s}_1^i[1] = 1$. The expected shares conditional on the individual choosing to visit all three sites are equivalent to the unconditional expected

desired shares; i.e., $c\hat{s}_j^i[1,2,3] = \hat{s}_j^i, \forall j$. However, they differ if the individual is assumed to visit only two of the sites. In these cases, the expected conditional shares are still calculated using equation (16) but with the set of included sites, L , appropriately restricted. For example, if the individual is assumed to visit only sites 1 and 3, the expected conditional shares, $c\hat{s}_j^i[1,3]$, are calculated using equation (16) but with $L = \{1,3\}$. Note that in this case, the shares generated by equation (16) are expected conditional shares rather than expected desired shares because conditioning on the choice set incorporates the truncation from desired to observed shares.

One can use these expected conditional shares to predict how individuals in different consumption groups will respond to small changes in the costs and catch rates. For example, for those individuals who are currently visiting only sites 1 and 3, a one percent increase in the price of site $j, j=1,2,3$ will lead to a $.461 * c\hat{s}_j^i$ increase in the individual's expected conditional share for site $m, m = 1,2,3, m \neq j$. However, a change in the price of site 2 will have no effect on the expected conditional shares for sites 1 and 3. While useful, the elasticities of the conditional shares must be interpreted with care because the individual is not constrained to stay in a particular group when catch rates and prices change. Therefore, a small change in either could cause a discrete jump from one group of sites to another.

IV. Conclusions and Areas of Future Research

This paper has analyzed a behavioral model that explicitly deals with corner solutions in demand analysis. The model is general enough to accommodate situations where one or more commodities is not consumed, and where only one of a set of commodities is consumed. Stochastic assumptions consistent with the

economic model were made and the likelihood function was derived. Parameters of a utility function were estimated from a recreation choice data set.

The key feature in the proposed behavioral model is that utility maximization is a two-step procedure. In the first step individuals maximize a function disregarding any constraints on the minimum amounts of commodities that can be consumed, and then in the second step eliminate from their budget those desired shares that are too small, and reallocate funds to the remaining commodities. This can be an optimal procedure in situations where optimization is costly.

The simplest form of the minimum quantity of a commodity constraint was employed in this paper, that of an absolute minimum, the same for each commodity. Other rules are possible, and should be investigated especially in situations where one has some prior reason to believe a particular mechanism is at work in the data. The mechanics of the likelihood need to be generalized to cases with more than three goods, and some rules of thumb developed for when considerations such as those of this paper are worth the additional computation time and expense. That is, it would be helpful if some guidelines were available, established perhaps by Monte Carlo methods, to determine for which kinds of data sets (considering especially the proportion of limit observations), these techniques will be necessary. These are topics of future research.

This research should be valuable to those interested in estimating demand systems from disaggregated data sets where a significant proportion of the observations involve consumption of some, but not all, of the available commodities. This is the case with many microeconomic data sets. The benefit of resulting parameter estimates is that they will have come from a plausible economic model with consistent stochastic assumptions.

References

- Abramowitz, M. and Stegun, I.A., eds. 1964: Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. U.S. Department of Commerce, National Bureau of Standards Applied Mathematics Series, 55.
- Amemiya, T. 1974: "Multivariate Regression and Simultaneous Equation Models When the Dependent Variables are Truncated Normal," Econometrica 42, 999-1012.
- Bocksteal, N.E., Hanemann W.M. and Strand I.E. 1984: Measuring the Benefits of Water Quality Improvements Using Recreational Demand Models, Vol. II. Report for Office of Policy Analysis, U.S. Environmental Protection Agency, Washington, D.C.
- Dubin, J.A. and McFadden, D.L. 1984: "An Econometric Analysis of Residential Electric Appliance Holding and Consumption," Econometrica 52, No. 2, 345-362.
- Hanneman, W.M. 1984: "Discrete Continuous Models and Consumer Demand," Econometrica 52, 541-561.
- Lee, L.F. and Pitt, M.M. 1986: "Microeconomic Demand Systems with Binding Non-Negativity Constraints: The Dual Approach," Econometrica 54, 1237-1242.
- Lee, L.F. and Pitt, M.M. 1987: "Microeconomic Models of Rationing, Imperfect Markets, and Nonnegativity Constraints," Journal of Econometrics 36, 89-110.
- Morey, E.R. 1981: "The Demand for Site-Specific Recreational Activities: A Characteristics Approach," Journal of Environmental Economics and Management 8, 345-371.
- Morey, E.R. 1984: "The Choice of Ski Areas: Estimation of a Generalized CES Preference Ordering with Characteristics," The Review of Economics and Statistics 56, No. 4.
- Morey, E.R. and Shaw, W.D. 1989: "An Economic Model to Assess the Impact of Acid Rain: A Characteristics Approach to Estimating the Demand for and Benefits from Recreational Fishing," in Advances in Applied Microeconomics: Vol 5, (eds., Link and Smith).
- Tobin, J. 1958: "Estimation of Relationships for Limited Dependent Variables," Econometrica 26, 24-36.
- Wales, T.J. and Woodland A.D. 1983: "Estimation of Consumer Demand System with Binding Non-Negativity Constraints," Journal of Econometrics 21, 263-285.
- Woodland A.D. 1979: "Stochastic Specification and the Estimation of Share Equations," Journal of Econometrics 10, 361-383.

APPENDIX

Let

$$w = ct \Rightarrow t = \frac{w}{c} \text{ and } dt = \frac{dw}{c}.$$

Recall that $c = 1/s_1$, then the limits of integration in equation (14) become

$$(A.1) \quad (1-s_m)s_1 \leq t \leq s_1 \Rightarrow (1-s_m)s_1 \leq \frac{w}{c} \leq s_1 \Rightarrow (1-s_m) \leq w \leq 1$$

The integral of (14) is written:

$$(A.2) \quad \int_{(1-s_m)s_1}^{s_1} t^{(\hat{x}_1+\hat{x}_2-2)} (1-ct)^{(\hat{x}_3-1)} dt = \int_{(1-s_m)}^1 \left(\frac{w}{c}\right)^{(\hat{x}_1+\hat{x}_2-2)} (1-w)^{(\hat{x}_3-1)} \left(\frac{dw}{c}\right) \\ = s_1^{(\hat{x}_1+\hat{x}_2-1)} \int_{1-s_m}^1 w^{(\hat{x}_1+\hat{x}_2-2)} (1-w)^{(\hat{x}_3-1)} dw$$

Thus:

$$(A.3) \quad f(D^*) = kS \left(\frac{s_2}{s_1}\right)^{(\hat{x}_2-1)} s_1^{(\hat{x}_1+\hat{x}_2-1)} \int_{(1-s_m)}^1 w^{(\hat{x}_1+\hat{x}_2-2)} [1-w]^{(\hat{x}_3-1)} dw$$

Simplifying the constant terms

$$kS \left(\frac{s_2}{s_1}\right)^{(\hat{x}_2-1)} s_1^{(\hat{x}_1+\hat{x}_2-1)} = k s_1^{(\hat{x}_1-1)} s_2^{(\hat{x}_2-1)} \sqrt{\frac{s_2}{s_1+s_2}} \stackrel{\text{def.}}{=} k_{12}$$

Thus (A.3) becomes

$$(A.4) \quad f(D^*) = k_{12} \int_{1-s_m}^1 w^{(\hat{x}_1+\hat{x}_2-2)} (1-w)^{(\hat{x}_3-1)} dw$$

A final transformation of variables is performed. Let

$$u = 1-w \Rightarrow w = 1-u \Rightarrow dw = -du$$

Then $1-s_m < w < 1 \Rightarrow 0 < u < s_m$

Thus,

$$(A.5) \quad f(D^*) = k_{12} \int_0^{s_m} (1-u)^{(\hat{x}_1+\hat{x}_2-2)} u^{(\hat{x}_3-1)} du$$

Equation (A.5) can be viewed as an incomplete beta function, i.e., the cumulative distribution of the beta function. The incomplete beta form of (A.5) is

$$(A.6) \quad I_{s_m}(\hat{x}_3, \hat{x}_1+\hat{x}_2-1) = \frac{1}{B(\hat{x}_3, \hat{x}_1+\hat{x}_2-1)} \int_0^{s_m} u^{(\hat{x}_3-1)} (1-u)^{(\hat{x}_1+\hat{x}_2-2)} du$$

By symmetry

$$I_{s_m}(\hat{x}_3, \hat{x}_1+\hat{x}_2-1) = 1 - I_{1-s_m}(\hat{x}_1+\hat{x}_2-1, \hat{x}_3)$$

Following Abramowitz and Stegun (1964, 26.5.16) I_{s_m} is written in

recurrence form as:

$$(A.7) \quad I_{s_m}(\hat{x}_3, \hat{x}_1+\hat{x}_2-1) = \frac{\Gamma(\hat{x}_1+\hat{x}_2+\hat{x}_3-1)}{\Gamma(\hat{x}_3+1)\Gamma(\hat{x}_1+\hat{x}_2-1)} s_m^{\hat{x}_3} (1-s_m)^{(\hat{x}_1+\hat{x}_2-1)} \\ + I_{s_m}(\hat{x}_3+1, \hat{x}_1+\hat{x}_2-1)$$

Thus, by symmetry

$$(A.8) \quad 1 - I_{1-s_m}(\hat{x}_1+\hat{x}_2-1, \hat{x}_3) = \left[1 - \frac{\Gamma(\hat{x}_1+\hat{x}_2+\hat{x}_3-1)}{\Gamma(\hat{x}_1+\hat{x}_2-1)\Gamma(\hat{x}_3+1)} (1-s_m)^{(\hat{x}_1+\hat{x}_2-1)} \right. \\ \left. \cdot s_m^{(\hat{x}_3-1)} - I_{1-s_m}(\hat{x}_1+\hat{x}_2, \hat{x}_3) \right]$$

Equation (A.6) can be rewritten as:

$$(A.9) \quad \int_0^{s_m} u^{(\hat{x}_3-1)} (1-u)^{(\hat{x}_1+\hat{x}_2-2)} du = B(\hat{x}_3, \hat{x}_1+\hat{x}_2-1) [I_{s_m}(\hat{x}_3, \hat{x}_1+\hat{x}_2-1)]$$

or by symmetry

$$\begin{aligned}
(A.10) \quad \int_0^{s_m} u^{(\hat{x}_3-1)} (1-u)^{(\hat{x}_1+\hat{x}_2-2)} du &= B(\hat{x}_3, \hat{x}_1+\hat{x}_2-1) [1 - I_{1-s_m}(\hat{x}_3, \hat{x}_1+\hat{x}_2-1)] \\
&= B(\hat{x}_3, \hat{x}_1+\hat{x}_2-1) \left[1 - \frac{\Gamma(\hat{x}_1+\hat{x}_2+\hat{x}_3-1)}{\Gamma(\hat{x}_2+\hat{x}_3)\Gamma(\hat{x}_3)} (1-s_m)^{(\hat{x}_1+\hat{x}_2-1)} \right. \\
&\quad \left. s_m^{\hat{x}_3} - I_{1-s_m}(\hat{x}_1+\hat{x}_2, \hat{x}_3) \right]
\end{aligned}$$

Finally,

$$\begin{aligned}
(A.11) \quad f(D^*) &= k_{12} \left\{ \frac{\Gamma(\hat{x}_1+\hat{x}_2+\hat{x}_3-1)}{\Gamma(\hat{x}_3)\Gamma(\hat{x}_1+\hat{x}_2-1)} \left[1 - \frac{\Gamma(\hat{x}_1+\hat{x}_2+\hat{x}_3-1)}{\Gamma(\hat{x}_1+\hat{x}_2)\Gamma(\hat{x}_3)} \right. \right. \\
&\quad \left. \left. - (1-s_m)^{(\hat{x}_1+\hat{x}_2-1)} s_m^{\hat{x}_3} - I_{1-s_m}(\hat{x}_1+\hat{x}_2, \hat{x}_3) \right] \right\} \\
&= k_{12} \left\{ B(\hat{x}_3, \hat{x}_1+\hat{x}_2-1) \left[1 - \frac{\Gamma(\hat{x}_1+\hat{x}_2+\hat{x}_3-1)}{\Gamma(\hat{x}_1+\hat{x}_2)\Gamma(\hat{x}_3)} \right. \right. \\
&\quad \left. \left. - (1-s_m)^{\hat{x}_1+\hat{x}_2-1} s_m^{\hat{x}_3} - I_{1-s_m}(\hat{x}_1+\hat{x}_2, \hat{x}_3) \right] \right\}
\end{aligned}$$

Steps to derive the likelihood functions of the other two boundary solutions $f(B^*)$ and $f(H^*)$ are similar to the above procedure used to find $f(D^*)$.

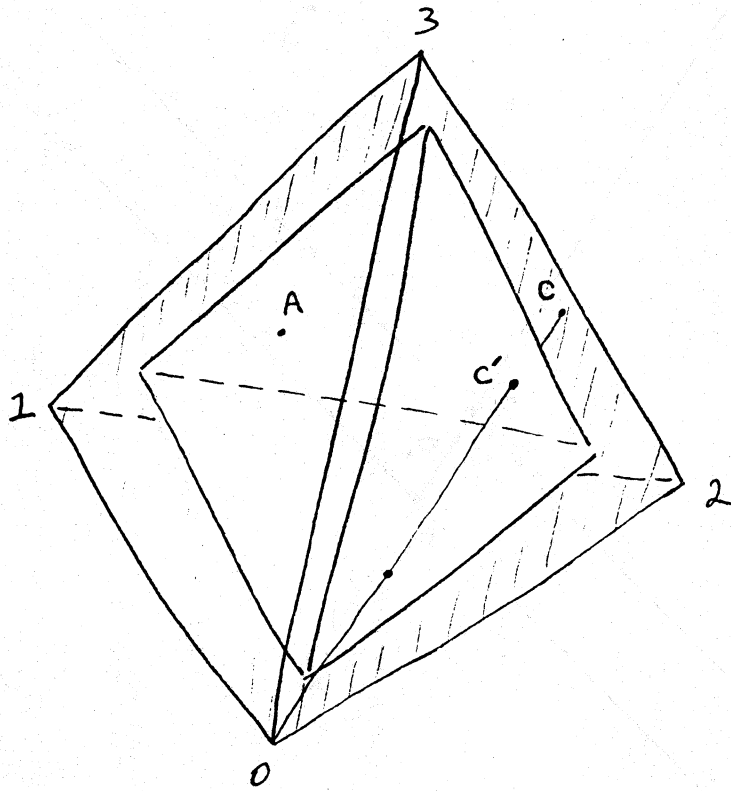


FIGURE 1

Figure 2 Graphical Representation of Corners, Boundary and Interior Solutions

